

數學二離散數學 2022 秋, 第一次期中考

學號: _____, 姓名: _____

本次考試共有 11 頁 (包含封面), 有 10 題。如有缺頁或漏題, 請立刻告知監考人員。

考試須知:

- 請在第一及最後一頁填上姓名學號。
- 不可翻閱課本或筆記。
- 計算題請寫出計算過程, 閱卷人員會視情況給予部份分數。
沒有計算過程, 就算回答正確答案也不會得到滿分。
答卷請清楚乾淨, 儘可能標記或是框出最終答案。

高師大校訓: 誠敬宏遠

誠, 一生動念都是誠實端正的。 敬, 就是對知識的認真尊重。
宏, 開拓視界, 恢宏心胸。 遠, 任重致遠, 不畏艱難。

請尊重自己也尊重其他同學, 考試時請勿東張西望交頭接耳。

1. (10 points) Find integers a and b such that

$$m^2 = a \binom{m}{2} + b \binom{m}{1}$$

for all m . Then use it to find the sum the series $1^2 + 2^2 + \dots + n^2$.

Answer: $a =$ _____, $b =$ _____, $1^2 + 2^2 + \dots + n^2 =$ _____.

2. (10 points) A bakery sells six different kinds of donuts, includes original, glazed, chocolated, banana milk, mango and strawberry. If the bakery has at least a dozen of each kind, how many different options for a dozen of donuts are there? What if a box is to contain at least three of chocolated donuts and two strawberry donuts?

一家麵包店銷售六種不同的甜甜圈，口味是原味、糖霜、巧克力、香蕉牛奶、芒果跟草莓。如果該店每種糕點至少有一打（12 個），那麼 (a) 可能配製成多少打不同種類的甜甜圈？(b) 如果盒子裡至少要有三個巧克力口味和兩個草莓口味的，又是多少種？

Answer: (a) _____, (b) _____.

3. (10 points) Twenty-four different books are to be put on six book shelves, each of which holds at least twenty-four books. 要將 24 本不同的書放到 6 個書架上，每個書架至少能放 24 本。
- (a) How many different arrangements are there if you only care about the number of books on the shelves (and not which book is where)? 如果只關心書架上書的數量（而不關心哪本書在哪個書架上），那麼有多少種不同的擺法？
- (b) How many different arrangements are there if you care about which books are where, but the order of the books on the shelves doesn't matter? 如果關心哪本書放在哪個書架上，但不關心書在書架上的順序，那麼有多少種不同的擺法？
- (c) How many different arrangements are there if the order on the shelves does matter? 如果需要考慮書在書架上的順序，那麼又有多少種不同的擺法？

Answer: (a) _____, (b) _____, (c) _____.

4. (10 points) A student has 37 days to prepare for an examination. From past experience she knows that she will require no more than 60 hours of study. She also wishes to study at least 1 hour per day. Show that no matter how she schedules her study time (a whole number of hours per day, however), there is a succession of days during which she will have studied exactly 13 hours.

一個學生有 37 天用來準備考試。根據以往的經驗，她知道她的唸書時間不超過 60 小時，她也希望她每天至少要唸一個小時的書。證明：無論她怎麼安排她的讀書計畫（每天的唸書時間都是一個整數小時），都存在連續的某幾天，在這幾天裡，她恰好唸了 13 個小時的書。

5. (10 points) Construct a permutation whose inversion sequences are 3, 2, 1, 2, 0, 0, 2, 0, 0.

Answer: _____.

6. (10 points) How many permutations of 1, 2, 3, 4, 5, 6, 7 have (a) exactly 23 inversions? (b) exactly 21 inversions? (c) exactly 19 inversions?

Answer: (a) _____, (b) _____, (c) _____.

7. (10 points) Determine the coefficient of $x_1^3 x_2 x_3^4 x_5^2$ in the expansion of

在下列展開式中找出 $x_1^3 x_2 x_3^4 x_5^2$ 這項的係數。

(a) $(x_1 + x_2 + x_3 + x_4 + x_5)^{10}$

(b) $(x_1 - x_2 + 3x_3 + x_4 - 2x_5)^{10}$

Answer: (a) _____, (b) _____.

8. (10 points) Prove that if 11 integers are selected from among $1, 2, \dots, 20$, then the selection includes integers a and b such that $a - b = 2$.

證明從 $1, 2, \dots, 20$ 中選定 11 個數字，則必定可以從選定的數中找出兩個數 a 跟 b 使得 $a - b = 2$ 。

9. (10 points) Use combinatorial reasoning to prove the identity

$$\binom{n}{n_1, n_2, \dots, n_k} = \binom{n-1}{n_1-1, n_2, \dots, n_k} + \binom{n-1}{n_1, n_2-1, \dots, n_k} + \dots + \binom{n-1}{n_1, n_2, \dots, n_k-1}$$

10. (10 points) Find the Ramsey number $R(5, 2)$ and explain why.

Answer: _____.

學號: _____, 姓名: _____, 以下由閱卷人員填寫

[illegible]