應數二離散數學 2023 春, 第一次期中考

學號: ________________________, 姓名: ________________________

本次考試共有 10 頁 (包含封面)，有 12 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一及最後一頁填上姓名學號。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。
  沒有計算過程，就算回答正確答案也不會得到滿分。
  答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：**誠敬宏遠**

誠，一生動念都是誠實端正的。 **敬**，就是對知識的認真尊重。
宏，開拓視界，恢宏心胸。 **遠**，任重致遠，不畏艱難。

請尊重自己也尊重其他同學，考試時請勿東張西望交頭接耳。

1. (5 points) Determine the coefficient of $x_1^3 x_2 x_3^5 x_5^3$ in the expansion of

   (a) $(x_1 + x_2 + x_3 + x_4 + x_5)^{12}$

   (b) $(x_1 - 2x_2 + 3x_3 + x_4 - 3x_5)^{12}$

   Answer: (a) ____________________, (b)____________________.

2. (10 points) How many ways can n couples (2n people) be seated in a line so that forno couple do the two people of the couple sit together. (All couples mustbe split up.)

   Answer: ____.

3. (10 points) A group of mn people are to be arranged into m teams each with n players.

   (a) Determine the number of ways if each team has a different name.

   (b) Determine the number of ways if the teams don't have names.

   Answer: (a) ______________________, (b) ______________________.

4. (10 points) A bakery sells five different kinds of donuts, includes original, glazed, chocolated, mango and strawberry. If the bakery has at least two dozens of each kind, how many different options for twenty of donuts are there? What if a box is to contain at least five of chocolated donuts and two strawberry donuts?

   Answer: (a) ______________________, (b) ______________________.

5. (10 points) Every day a student walks from her home to school, which is located 10 blocks east and 14 blocks north from home. She always takes a shortest walk of 24 blocks.

   (a) How many different walks are possible? 有多少種不同的步行方式？

   (b) Suppose that five blocks east and four blocks north of her home lives her best friend, whom she meets each day on her way to school. Now how many different walks are possible?

   (c) Suppose, in addition, that three blocks east and five blocks north of her friend's house there is a park where the two girls stop each day to rest and play. Now how many different walks are there?

   (d) Stopping at a park to rest and play, the two students often get to school late. To avoid the temptation of the park, our two students decide never to pass the intersection where the park is. Now how many different walks are there?

   Answer: (a) ____________, (b) ____________, (c) ____________, (d) ____________.

6. (10 points) A student has 35 days to prepare for an examination. From past experience she knows that she will require no more than 53 hours of study. She also wishes to study at least 1 hour per day. Show that no matter how she schedules her study time (a whole number of hours per day, however), there is a succession of days during which she will have studied exactly 16 hours.

7. (10 points) Determine the mobile integers in

$$\overrightarrow{4}\ \overleftarrow{8}\ \overrightarrow{3}\ \overleftarrow{1}\ \overrightarrow{6}\ \overleftarrow{7}\ \overleftarrow{2}\ \overrightarrow{5}$$

and then use the algorithm for generating the next six permutations.

Answer: the mobile integers are ____________________.

(1) ____________________. (2) ____________________.

(3) ____________________. (4) ____________________.

(5) ____________________. (6) ____________________.

8. (10 points) Construct a permutation whose inversion sequences are 6, 2, 4, 3, 0, 1, 2, 0, 0.

   Answer: ____.

9. (10 points) Determine the number of integral solutions of the equation

$$x + y + z + w = 25$$

that satisfy

$$3 \le x \le 8, \ -1 \le y \le 6, \ 5 \le z \le 9, \ 5 \le w \le 9.$$

Answer: ____.

10. (10 points) Determine the number of ways to place six non-attacking rooks on the following 6-by-6 board, with forbidden positions as shown.

x	x				
	x	x			
		x			
				x	x
				x	x

Answer: ____________________________.

11. (10 points) A collection of subsets of $1, 2, ..., n$ has the property that each pair of subsets has at least one element in common. Prove that there are at most 2^{n-1} subsets in the collection.

12. (10 points) Use combinatorial reasoning (組合解釋) to prove the identity (in the form given)

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}.$$

(Hint: Let S be a set with three distinguished elements a, b and c and count certain k-subsets of S.)

學號: ____________, 姓名: ____________, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	5	10	10	10	10	10	10	10	10	10	10	10	115
Score:													