數學二離散數學 2025 秋, 第一次期中考 解答

學號: ________________________________, 姓名: ________________________________

本次考試共有 10 頁 (包含封面)，有 13 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一及最後一頁填上姓名學號。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。
  沒有計算過程，就算回答正確答案也不會得到滿分。
  答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：**誠敬宏遠**

誠，一生動念都是誠實端正的。 **敬**，就是對知識的認真尊重。
宏，開拓視界，恢宏心胸。 **遠**，任重致遠，不畏艱難。

請尊重自己也尊重其他同學，考試時請勿東張西望交頭接耳。

1. (5 points) Find the sum

$$\sum_{k=0}^{n} \binom{n}{k} r^k$$

for all n.

Answer: $(r+1)^n$

Solution：

Ch. 5 problem 7.

Theorem 5.2.2, with x=r.

2. (5 points) Determine the coefficient of $x_1^3 x_2 x_3^4 x_5^2$ in the expansion of

$$(3x_1 - 2x_2 + x_3 + x_4 - x_5)^{10}.$$

Answer: $\binom{10}{3,1,4,0,2} \times 3^3 \times (-2) \times 1^4 \times (-1)^2 = -680400$

Solution：

Similar with Ch. 5 problem 39.

Theorem 5.4.1, with $y_1 = 3x_1,\ y_2 = -2x_2,\ y_3 = x_3, y_4 = x_4,\ y_5 = -x_5$.

3. (10 points) In how many ways can 13 indistinguishable apples and 2 orange be distributed among three children in such a way that each child gets at least one piece of fruit?

Answer: $\binom{3}{1}\binom{11+3-1}{3-1}+\binom{3}{2}\binom{12+3-1}{3-1}=507$.

Solution：

Case 1: hand out 2 orange for one child:

stage	to do	# choices
1	hand out 2 orange for one child	$\binom{3}{1}$
2	give one apple to each of the other children	1
3	distribute remaining 11 apples to 3 children	$\binom{11+3-1}{3-1}$

Case 2: hand out 2 orange for two children:

stage	to do	# choices
1	hand out 2 orange for two children	$\binom{3}{2}$
2	give one apple to each of the other children	1
3	distribute remaining 12 apples to 3 children	$\binom{12+3-1}{3-1}$

Answer: $\binom{3}{1}\binom{11+3-1}{3-1}+\binom{3}{2}\binom{12+3-1}{3-1}$

4. (10 points) Assume there is a standard deck of 52 cards. a) How many cards must be selected to guarantee that at least three cards of the same suit are chosen? b) How many cards must be selected to guarantee that at least three hearts are selected

Answer: (a) **4*2+1=9** , (b) **13*3+2+1=42** .

Solution：

一副標準的 52 張撲克牌，組成為四個花色（黑桃、紅心、方塊、梅花），每種花色有 13 個不同的點數。

(a) 沒有三張相同的花色 $\Rightarrow$ 四種花色，每種最多兩張 $=2\times 4=8$。

(b) 沒有三張紅心 $\Rightarrow$ 紅心最多兩張，其他三種花色最多各 13 張。

5. (10 points) Given 200 integers $a_1, a_2, ..., a_{200}$, there exist integers r and s with $0 \leq r < s \leq 200$ such that $a_{r+1} + a_{r+2} + ... + a_s$ is divisible by 200. Less formally, there exist consecutive a's in the sequence $a_1, a_2, ..., a_{200}$ whose sum is divisible by m.

Solution：

Similar to textbook page 71, section 3.1 application 3.

6. (10 points) In which position does the subset 2579 occur in the lexicographic order of the 4-subsets of {1, 2, 3, 4, 5, 6, 7, 8, 9}?

Answer: **86** .

Solution：

By textbook Theroem 4.4.2，$n = 9, r = 4, a_1 = 2, a_2 = 5, a_3 = 7, a_4 = 9$。

$$
\begin{aligned}
&\binom{9}{4} - \binom{9-2}{4} - \binom{9-5}{3} - \binom{9-7}{2} - \binom{9-9}{1} \\
=&\binom{9}{4} - \binom{7}{4} - \binom{4}{3} - \binom{2}{2} - \binom{0}{1} \\
=&\frac{9!}{4!5!} - \frac{7!}{4!3!} - 4 - 1 - 0 \\
=&86
\end{aligned}
$$

7. (10 points) Determine the number of integral solutions of the equation

$$x + y + z + w = 22$$

that satisfy

$$3 \leq x \leq 6, \ \ -7 \leq y \leq 0, \ 2 \leq z \leq 9, \ 6 \leq w \leq 9.$$

Answer: $\binom{21}{3} - [\binom{17}{3} + \binom{13}{3}] \times 2 + [\binom{9}{3} \times 4 + \binom{5}{3} + \binom{13}{3}] - [\binom{5}{3} \times 2] = 10$.

8. (10 points) Compute the 6th derangement number D_6 in any way you can.

Solution：

我記錯範圍了，送分！

9. (5 points) Construct a permutation whose inversion sequences are 5, 3, 4, 5, 1, 3, 1, 0, 0.

   Answer: **857291364** .

10. (5 points) What bit 8-tuple immediately follows 10011000 in the reflected Gray code scheme?.

    Answer: **10001000**

11. (10 points) Show that if n + 1 distinct integers are chosen from the set {l, 2, ... , 5n}, then there are always two which differ by at most 4.

Solution：

Ch 3, problem 5.

12. (10 points) Show that an m-by-n chessboard has a perfect cover by dominoes if and only if at least one of m and n is even.

Solution：

Ch1 problem 1, 上課有證！

13. (10 points) Use combinatorial reasoning (組合解釋) to prove the identity.

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$$

p.s. 代數證明全對給 3 分數，沒有部份分。

Solution：

Ch 5, eq. (5.16) 上課有證！

學號: ____________, 姓名: ____________, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Points:	5	5	10	10	10	10	10	10	5	5	10	10	10	110
Score:														