姓名: Sol. 葉均承 應數一線性代數

學號: ______ **Quiz 10** 考試日期: 2020/06/11

不可使用手機、計算器，禁止作弊!
背面還有題目

1. (50 points) Find all $a, b \in \mathbb{C}$ such that the matrix $\underbrace{\begin{bmatrix} i & a \\ b & i \end{bmatrix}}_{A}$ is unitarily diagonalizable.

$\Updownarrow$ normal

i.e. $AA^* = A^*A$

$$AA^* = \begin{bmatrix} i & a \\ b & i \end{bmatrix}\begin{bmatrix} -i & \bar{b} \\ \bar{a} & -i \end{bmatrix}$$

$$= \begin{bmatrix} 1 + a\bar{a} & \bar{b}i - ai \\ -bi + \bar{a}i & 1 + b\bar{b} \end{bmatrix}$$

$$= A^*A = \begin{bmatrix} 1 + b\bar{b} & -ai + \bar{b}i \\ \bar{a}i - bi & 1 + a\bar{a} \end{bmatrix}$$

$\Rightarrow a\bar{a} = b\bar{b}$, with $a\bar{a} = |a|^2$ and $b\bar{b} = |b|^2$, 又$\because |x| \geq 0$, $\forall x \in \mathbb{C}$ ~~∀x∈~~

$\therefore \boxed{|a| = |b|}$

2. (50 points) Mark all the matrix if it is a Jordan Canonical form and boxed all the Jordan blocks in it.

Yes / No (a) $\begin{bmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

Yes / No (b) $\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

Yes / No (c) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Yes / No (d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$

Yes / No (e) $\begin{bmatrix} i & 1 & 0 & 0 \\ 0 & i & 1 & 0 \\ 0 & 0 & i & 1 \\ 0 & 0 & 0 & i \end{bmatrix}$