姓名: ______________ 葉均承 應數一線性代數

學號: ______________ **Quiz 1** 考試日期: 2020/03/19

不可使用手機、計算器，禁止作弊!
背面還有題目

1. (50%) Let

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

Find (if exists) an invertible matrix C and a diagonal matrix D such that $D = C^{-1}AC$. Also, find the eigenvalues of A^5.

Is A diagonalizable? Yes

If so, eigenvalues of A^5 are: $2^5, 1^5$, C= $\begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, and D= $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

$$|A-\lambda I| = \begin{vmatrix} -\lambda & 0 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{vmatrix} = -\lambda(2-\lambda)(3-\lambda) - (-2)(2-\lambda)$$

$$= (2-\lambda)\left[\lambda^2 - 3\lambda + 2\right] = (\lambda-1)(\lambda-2)^2$$

$\therefore \lambda = 1, 2$

$\boxed{\lambda = 1}$

$$\begin{bmatrix} -1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

let $z = r$

$$\begin{cases} x + 2z = 0 \\ y - z = 0 \end{cases}$$

$\therefore x = -2r,\ y = r$

$$V = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} r$$

$\boxed{\lambda = 2}$

$$\begin{bmatrix} -2 & 0 & -2 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

let $y = r,\ z = s$

$x + z = 0$

$\Rightarrow x = -s$

$$V = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} s + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} r$$

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2. (50%)Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}$$

Find (if exists) an invertible matrix C and a diagonal matrix D such that $D = C^{-1}AC$. Also, find the eigenvalues of A^5.

Is A diagonalizable?__No__

If so, eigenvalues of A^5 are:______, C= ________________, and D=________________.

$$|A-\lambda I| = \begin{vmatrix} \lambda-1 & 0 & 0 \\ 1 & 2-\lambda & 0 \\ -3 & 5 & 2-\lambda \end{vmatrix} = (\lambda-1)(2-\lambda)^2$$

$\therefore \lambda = 1 , 2$

$\boxed{\lambda=1}$

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ -3 & 5 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ -3 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 8 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

let $z = r$

$\begin{cases} x+y=0 \\ 8y+z=0 \end{cases}$

$y = \frac{-1}{8}r , x = \frac{1}{8}r$

$V = \begin{bmatrix} \frac{1}{8} \\ \frac{-1}{8} \\ 1 \end{bmatrix} r$

$\boxed{\lambda=2}$

$$\begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ -3 & 5 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ -3 & 5 & 0 \\ -1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

let $z = r$

$\begin{cases} x=0 \\ 5y=0 \end{cases}$

$V = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$

$\therefore$ the algebraic multiplicity of eigenvalue 2 is 2.

the geometric multiplicity of eigenvalue 2 is 1.

$> 1 \neq 2$

$\therefore$ A is NOT diagonalizable!