姓名: SOLUTION　　　　　　　　　　葉均承　　應數一線性代數

學號: ________　　　**Quiz 3**　　　考試日期: 2020/04/09

不可使用手機、計算器，禁止作弊!
背面還有題目

1. Find the projection of [2, 4, 2] on the subspace $W = \text{sp}([2, 1, 1], [1, 0, 1])$ in $\mathbb{R}^3$

   Answer: $[\frac{10}{3}, \frac{8}{3}, \frac{2}{3}] = \frac{2}{3}[5, 4, 1]$

   Let $\vec{b} = [2, 4, 2], \vec{v}_1 = [2, 1, 1], \vec{v}_2 = [1, 0, 1]$

$$A = \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

   Let $x_3 = r$, $\begin{cases} 2x_1 + x_2 + x_3 = 0 \\ \quad\;\; - x_2 + x_3 = 0 \end{cases}$, we have $[x_1, x_2, x_3] = [-r, r, r]$.
   Therefore, $W^\perp = sp([-1, 1, 1])$.

   method 1

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 2 \\ 1 & 0 & 1 & 4 \\ 1 & 1 & 1 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & -1 & 2 \\ 0 & -1 & 3 & 6 \\ 0 & 1 & 3 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & -1 & 2 \\ 0 & 1 & -3 & -6 \\ 0 & 1 & 3 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & -1 & 2 \\ 0 & 1 & -3 & -6 \\ 0 & 0 & 6 & 8 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 2 & 1 & -1 & 2 \\ 0 & 1 & -3 & -6 \\ 0 & 0 & 1 & 4/3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & 0 & 10/3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4/3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 0 & 0 & 16/3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4/3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 8/3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4/3 \end{array}\right]$$

   Hence, $\vec{b}_W = \frac{8}{3}\vec{v}_1 - 2\vec{v}_2 = \frac{8}{3}[2, 1, 1] - 2[1, 0, 1] = [\frac{10}{3}, \frac{8}{3}, \frac{2}{3}]$

   method 2

$$\vec{b}_{W^\perp} = \frac{\vec{b} \cdot \vec{v}_3}{\|\vec{v}_3\|^2}\vec{v}_3 = \frac{[2, 4, 2] \cdot [-1, 1, 1]}{\|[-1, 1, 1]\|^2}[-1, 1, 1] = \frac{4}{3}[-1, 1, 1]$$

$$\vec{b}_W = \vec{b} - \vec{b}_{W^\perp} = [2, 4, 2] - \frac{4}{3}[-1, 1, 1] = [\frac{10}{3}, \frac{8}{3}, \frac{2}{3}]$$

2. Find the projection of [1, 2, 1] on the plane $x + 2y - z = 0$ in $\mathbb{R}^3$

   Answer: $[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}] = \frac{1}{3}[1, 2, 3]$

   Let W be the subspace of $\mathbb{R}^3$ given by the plane $x + 2y - z = 0$, and let $\vec{a} = [1, 2, -1]$.
   Then $W^\perp = sp(\vec{a}) = sp([1, 2, -1])$.
   Let $\vec{b} = [1, 2, 1]$, then

$$\vec{b}_{W^\perp} = \frac{\vec{b} \cdot \vec{a}}{\|\vec{a}\|^2}\vec{a} = \frac{[1, 2, 1] \cdot [1, 2, -1]}{\|[1, 2, -1]\|^2}[1, 2, -1] = \frac{4}{6}[1, 2, -1]$$

$$\vec{b}_W = \vec{b} - \vec{b}_{W^\perp} = [1, 2, 1] - \frac{2}{3}[1, 2, -1] = [\frac{1}{3}, \frac{2}{3}, \frac{5}{3}]$$