Quiz 4

不可使用手機、計算器，禁止作弊!
背面還有題目

1. (50%) Find an orthonormal basis for the plane $2x + y - z = 0$ in $\mathbb{R}^3$

   Answer: $[2, -5, -1]$

   Method 1

   The normal vector $\vec{n}$ of the plane is $[2, 1, -1]$

   Pick two points (0, 0, 0) and (1, 0, 2) in the plane, therefore, the vector $\vec{a} = [1, 0, 2]$ in the plane.

   Let $\vec{b} = \vec{a} \times \vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -1 \\ 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix}\vec{i} + \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}\vec{j} + \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}\vec{k} = [2, -5, -1]$

   $\{\vec{a}, \vec{b}\} = \{[1, 0, 2], [2, -5, -1]\}$ is an orthogonal basis for the plane.
   $\{\frac{1}{\sqrt{5}}[1, 0, 2], \frac{1}{\sqrt{30}}[2, -5, -1]\}$ is an orthonormal basis for the plane.

   Method 2

   The normal vector $\vec{n}$ of the plane is $[2, 1, -1]$.
   Note that $\{[2, 1, -1], [1, 0, 0], [0, 1, 0]\}$ is a basis for $\mathbb{R}^3$.
   Using Gram-Schmidt process to get $\{\frac{1}{3}[2, 1, -1], \frac{1}{3}[\frac{1}{3}, \frac{-1}{3}, \frac{1}{3}], \frac{1}{3}[0, \frac{1}{2}, \frac{1}{2}]\}$ is an orthonormal basis for $\mathbb{R}^3$.

   $\{\frac{1}{3}[\frac{1}{3}, \frac{-1}{3}, \frac{1}{3}], \frac{1}{3}[0, \frac{1}{2}, \frac{1}{2}]\}$ is an orthonormal basis for the plane.

2. (50%) Verify(驗證) that $\{[2,3,1],[-1,1,-1]\}$ is an orthogonal set, and $W = sp([2,3,1],[-1,1,-1])$. Given $\vec{b} = [2,1,4]$, find the projection of $\vec{b}$ on W. (i.e. find $\vec{b}_W$)

Verify: $[2,3,1]\cdot[-1,1,-1] = 0$ ...... ok! $\{[2,3,1],[-1,1,-1]\}$ is an orthogonal set

$$\begin{aligned}
\vec{b}_W &= \frac{\vec{b}\cdot\vec{v}_1}{\vec{v}_1\cdot\vec{v}_1}\,\vec{v}_1 + \frac{\vec{b}\cdot\vec{v}_2}{\vec{v}_2\cdot\vec{v}_2}\,\vec{v}_2 \\
&= \frac{[2,1,4]\cdot[2,3,1]}{[2,3,1]\cdot[2,3,1]}\,[2,3,1] + \frac{[2,1,4]\cdot[-1,1,-1]}{[-1,1,-1]\cdot[-1,1,-1]}\,[-1,1,-1] \\
&= \frac{11}{14}[2,3,1] + \frac{-5}{3}[-1,1,-1] \\
&= [\frac{136}{42},\frac{29}{42},\frac{103}{42}]
\end{aligned}$$