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葉均承　　應數一線性代數

學號: ________

Quiz 6

考試日期: 2020/05/13

不可使用手機、計算器，禁止作弊!
背面還有題目

1. (50%) Find the least-squares linear fit to the data points (-4, -2), (-2, 0), (0, 1), (2, 4), (4, 5)

linear function : $y = r_0 + r_1 x$

$$\begin{bmatrix} 1 & -4 \\ 1 & -2 \\ 1 & 0 \\ 1 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 1 \\ 4 \\ 5 \end{bmatrix}$$

$$A \quad \vec{r} = \vec{b}$$

$$\vec{r} = (A^TA)^{-1}A^T\vec{b}$$

$$= \begin{bmatrix} 1/5 & 0 \\ 0 & 1/40 \end{bmatrix} \begin{bmatrix} 8 \\ 36 \end{bmatrix} = \begin{bmatrix} 8/5 \\ 36/40 \end{bmatrix} = \begin{bmatrix} 8/5 \\ 9/10 \end{bmatrix}$$

$$\boxed{\therefore\ y = \frac{8}{5} + \frac{9}{10}x}$$

$$A^TA = \begin{bmatrix} 5 & 0 \\ 0 & 40 \end{bmatrix}$$

$$(A^TA)^{-1} = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{40} \end{bmatrix}$$

$$A^T\vec{b} = \begin{bmatrix} 8 \\ 36 \end{bmatrix}$$

2. (50%) Find the change-of-coordinates matrix from B to B' and from B' to B, indicate which is which.

$$B = (\underset{\vec{b}_1}{x^2}, \underset{\vec{b}_2}{x}, \underset{\vec{b}_3}{1}) \text{ and } B' = (\underset{\vec{b}'_1}{x^2 - x}, \underset{\vec{b}'_2}{2x^2 - 2x + 1}, \underset{\vec{b}'_3}{x^2 - 2x})$$

$$M_B = \begin{array}{c} \begin{matrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}, \quad M_{B'} = \begin{array}{c} \begin{matrix} \vec{b}'_1 & \vec{b}'_2 & \vec{b}'_3 \end{matrix} \\ \begin{bmatrix} 1 & 2 & 1 \\ -1 & -2 & -2 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

① $$\begin{cases} C_{B,B'} = M_{B'}^{-1} M_B \\ C_{B',B} = M_B^{-1} M_{B'} \end{cases}$$

$$\begin{cases} \left[M_{B'} \mid M_B \right] \sim \left[I \mid C_{B,B'} \right] \\ \left[M_B \mid M_{B'} \right] \sim \left[I \mid C_{B',B} \right] \end{cases}$$

Note: $$\begin{cases} \vec{V}_{B'} = C_{B,B'} \vec{V}_B \\ \vec{V}_B = C_{B',B} \vec{V}_{B'} \end{cases}$$

$$\left[M_{B'} \mid M_B \right] \sim \left[I \,\middle|\, \begin{matrix} 2 & 1 & -2 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{matrix} \right] \Rightarrow C_{B,B'} = \begin{bmatrix} 2 & 1 & -2 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$C_{B',B} = M_B^{-1} M_{B'} = M_{B'} = \begin{bmatrix} 1 & 2 & 1 \\ -1 & -2 & -2 \\ 0 & 1 & 0 \end{bmatrix}$$