姓名: sol.　　　　　　　　　　　　　　　　　　葉均承　　應數一線性代數

學號: ______　　　　**Quiz 7**　　　　考試日期: 2020/05/21

不可使用手機、計算器，禁止作弊!

1. Find the matrix representations $R_{B,B}$, $R_{B',B'}$ and an invertible C such that $R_{B',B'} = C^{-1}R_{B,B}C$ for the given linear transformation T.
$T : \mathbb{R}^3 \to \mathbb{R}^3$ defined as projection of $\mathbb{R}^3$ through the plane $x + y + z = 0$; $B = E$, $B' = ([1,0,-1], [1,-1,0], [1,1,1])$.

①　$T(\vec{V})_B = R_{B,B}\vec{V}_B$, ∵ $B=E$ ∴ $T(\vec{V})_B = T(\vec{V}) = A\vec{V}$

∴ $R_{B,B} = A$: the s.m.r of T.

T: projection through $x+y+z=0$

i.e. $\begin{cases} T(\vec{n}) = 0\times\vec{n}, & \text{where } \vec{n} = [1,1,1] \\ T(\vec{w}) = \vec{w}, & \text{where } \vec{w} \text{ in } x+y+z=0 \end{cases}$

Note $\{[1,0,-1],[1,-1,0]\}$ are ~~an orthogonal basis~~ a basis of $x+y+z=0$

$$\therefore A\underbrace{\begin{bmatrix}1&1&1\\0&-1&1\\-1&0&1\end{bmatrix}}_{C} = \underbrace{\begin{bmatrix}1&1&1\\0&-1&1\\-1&0&1\end{bmatrix}}_{C}\underbrace{\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}}_{D} \qquad \therefore \underset{R_{B,B}}{A} = CDC^{-1} = \frac{1}{3}\begin{bmatrix}2&-1&-1\\-1&2&-1\\-1&1&2\end{bmatrix}$$

②　$T(\vec{V})_{B'} = R_{B',B'}\vec{V}_{B'} = R_{B',B'} \times C_{B,B'}\vec{V}_B$

$\| $

$C_{B,B'}T(\vec{V})_B = C_{B,B'}R_{B,B}\vec{V}_B$

∴ $R_{B',B'}C_{B,B'} = C_{B,B'}R_{B,B}$

∴ $R_{B',B'} = C_{B,B'}R_{B,B}C_{B,B'}^{-1}$

(i) $C_{BB'} = M_{B'}^{-1}M_B$, ∵ $\left[\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&-1&1&0&1&0\\-1&0&1&0&0&1\end{array}\right] \sim \Big[\, I \;\Big|\; \underbrace{\begin{matrix}1/3&1/3&-2/3\\1/3&-2/3&1/3\\1/3&1/3&1/3\end{matrix}}_{C_{B,B'}}\Big]$

(ii) $C_{B,B'}^{-1} = C_{B',B} = M_B^{-1}M_{B'} = \begin{bmatrix}1&1&1\\0&-1&1\\-1&0&1\end{bmatrix}$

∴ $R_{B',B'} = C_{B,B'}R_{BB}C_{B,B'}^{-1} \overset{\text{by ①}}{=\!=} C^{-1}AC = D = \begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}$