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學號: ______ **Quiz 8** 考試日期: 2020/05/28

不可使用手機、計算器，禁止作弊!
背面還有題目

1. Find the modulus and principal argument for $(-\sqrt{3}-i) = z$

Write z into polar coordinate (r, θ), then r = modulus or magnitude or $|z|$
$-\pi < \theta \leq \pi$ θ = principal argument

$z = -\sqrt{3} - i$, $\therefore |z| = \sqrt{z\bar{z}} = \sqrt{(-\sqrt{3}-i)(-\sqrt{3}+i)} = \sqrt{(-\sqrt{3})^2 - i^2} = \sqrt{3+1} = \sqrt{4} = 2$.

$z = r\cos\theta + ir\sin\theta$ $\quad \therefore \begin{cases} \cos\theta = \frac{-\sqrt{3}}{2} \\ \sin\theta = -\frac{1}{2} \end{cases}$ $\quad \therefore \tan\theta = \frac{1}{\sqrt{3}}$ $\quad \therefore \theta = \frac{-5\pi}{6}$

$\therefore$ $\boxed{\begin{array}{l} \text{modulus: } 2 \\ \text{principal argument: } \frac{-5\pi}{6} \end{array}}$

2. Find the inner product $< \vec{u}, \vec{v} >$ (or $\vec{u} \cdot \vec{v}$) and , where $\vec{u} = [2+i, 2, i], \vec{v} = [i, 1+i, 2-i]$

$$< \vec{u}, \vec{v} > = \overline{\vec{u}}^T \vec{v} = \overline{[2+i, 2, i]} \begin{bmatrix} i \\ 1+i \\ 2-i \end{bmatrix} = [2-i, 2, -i] \begin{bmatrix} i \\ 1+i \\ 2-i \end{bmatrix}$$

$$= i(2-i) + (1+i)2 + (2-i)(-i)$$

$$= 2+2i$$

3. Find the six sixth roots of -125. $= -5^3 = z$

$\therefore \sqrt[6]{|-125|} = \sqrt[6]{125} = \sqrt{5}$

$-1 = \cos(\pi) + i\sin(\pi)$

$\therefore$ let $w = r(\cos\theta + i\sin\theta)$ s.t. $w^6 = z$

i.e. $w^6 = r^6(\cos(6\theta) + i\sin(6\theta)) = z = 5^3(\cos(\pi) + i\sin(\pi))$

$\therefore r = \sqrt{5}$

$6\theta = \pi \Rightarrow \theta = \frac{\pi}{6} + \frac{2k\pi}{6}$, $k = 0, 1, 2, 3, 4, 5$

$\therefore \theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}$

$\therefore w =$ ① $\sqrt{5}\left(\cos\frac{\pi}{6} + i\sin\left(\frac{\pi}{6}\right)\right) = \sqrt{5}\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$

② $\sqrt{5}\left(\cos\left(\frac{3\pi}{6}\right) + i\sin\left(\frac{3\pi}{6}\right)\right) = \sqrt{5}(0 + i)$

③ $\sqrt{5}\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right) = \sqrt{5}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

④ $\sqrt{5}\left(\cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right)\right) = \sqrt{5}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

⑤ $\sqrt{5}\left(\cos\left(\frac{9\pi}{6}\right) + i\sin\left(\frac{9\pi}{6}\right)\right) = \sqrt{5}(0 - i)$

⑥ $\sqrt{5}\left(\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right) = \sqrt{5}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$