姓名: SOLUTION 葉均承 應數一線性代數

學號: _____ **Quiz10** 考試日期: 2020/12/17

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊! 3. 背面還有題目

1. Let A be a 3×3 matrix with row vectors $\vec{a},\vec{b},\vec{c}$ and with determinant equal to 3. Find the determinant of the matrix having the row vectors as $\vec{a}+\vec{a},\vec{b}+\vec{c},\vec{c}+\vec{a}$

$$A=\begin{bmatrix} - & \vec{a} & - \\ - & \vec{b} & - \\ - & \vec{b} & - \end{bmatrix}, B=\begin{bmatrix} - & \vec{a}+\vec{a} & - \\ - & \vec{b}+\vec{c} & - \\ - & \vec{c}+\vec{a} & - \end{bmatrix}$$

Answer: $|B|=$ 6 .

$$\begin{aligned} |B| &= \begin{vmatrix} - & \vec{a}+\vec{a} & - \\ - & \vec{b}+\vec{c} & - \\ - & \vec{c}+\vec{a} & - \end{vmatrix} \\ &= \begin{vmatrix} - & 2\vec{a} & - \\ - & \vec{b}+\vec{c} & - \\ - & \vec{c}+\vec{a} & - \end{vmatrix} \rightsquigarrow R_1=\frac{1}{2}R_1 \\ &= 2\begin{vmatrix} - & \vec{a} & - \\ - & \vec{b}+\vec{c} & - \\ - & \vec{c}+\vec{a} & - \end{vmatrix} \rightsquigarrow R_3=R_3-R_1 \\ &= 2\begin{vmatrix} - & \vec{a} & - \\ - & \vec{b}+\vec{c} & - \\ - & \vec{c} & - \end{vmatrix} \rightsquigarrow R_2=R_2-R_3 \\ &= 2\begin{vmatrix} - & \vec{a} & - \\ - & \vec{b} & - \\ - & \vec{c} & - \end{vmatrix} \\ &= 2|A| \\ &= 6 \end{aligned}$$

2. Find the determinant of A

$$A = \begin{bmatrix} 4 & -1 & 1 & 0 \\ 2 & 6 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 3 & 1 & 2 & -1 \end{bmatrix}$$

Answer: $|A|=$ -6 .

$$|A| = \begin{vmatrix} 4 & -1 & 1 & 0 \\ 2 & 6 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 3 & 1 & 2 & -1 \end{vmatrix} \rightsquigarrow \text{ expand on the } 3^{rd} \text{ row.}$$

$$= (-1)^{2+3} \times 3 \times \begin{vmatrix} 4 & 1 & 0 \\ 2 & 0 & 0 \\ 3 & 2 & -1 \end{vmatrix} \rightsquigarrow \text{ expand on the } 2^{nd} \text{ row.}$$

$$= (-1)^{2+3} \times 3 \times (-1)^{1+2} \times 2 \times \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix}$$

$$= (-1)^{2+3} \times 3 \times (-1)^{1+2} \times 2 \times [1 \times (-1) - 0 \times 2]$$

$$= -6$$