姓名: Sol 葉均承 應數一線性代數

學號:

Quiz 1

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1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊! 3. 背面還有題目

1. (50%) Determine whether the vector $\vec{b}$ is in the span of the vectors $\vec{v_i}$. If so, write $\vec{b}$ into the linear combination form.

$$\vec{b} = \begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix}, \vec{v_1} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} -3 \\ -1 \\ 5 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & -3 & 3 \\ 2 & 4 & -1 & 5 \\ 4 & -2 & 5 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$\therefore$ Yes! $\vec{b} \in sp(\vec{V_1}, \vec{V_2}, \vec{V_3})$

$\vec{b} = 2\vec{V_1} + 0\vec{V_2} - \vec{V_3}$

2. (50%) Prove that the given relation holds for all vectors, matrices and scalars for which the expression are defined.

$$(AB)^T = B^T A^T$$

△ $A = [a_{ij}]$, $B = [b_{ij}]$, let $AB = C = [c_{ij}]$

Note $c_{ij} = \sum_k a_{ik} b_{kj}$ $\therefore (AB)^T = C^T = [c'_{ij}]$ $\therefore c'_{ij} = c_{ji} = \sum_k a_{jk} b_{ki}$

△ $A^T = [a'_{ij}]$, $B^T = [b'_{ij}]$, let $B^T A^T = D = [d_{ij}]$

$a'_{ij} = a_{ji}$ $b'_{ij} = b_{ji}$

$d_{ij} = \sum_k b'_{ik} a'_{kj} = \sum_k b_{ki} a_{jk} = \sum_k a_{jk} b_{ki} = c'_{ij}$

$\therefore C = D$