

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊! 3. 背面還有題目

1. (50%) Determine whether the given subset is a subspace of \mathbb{R}^3 . Please give reasons to support your answer.

$$\{[2x, x + y, y] | x, y \in \mathbb{R}\}$$

Circle the answer: (Yes / NO), and write your reason below.

solution 1:

For any $\vec{v}, \vec{u} \in W$, $r \in \mathbb{R}$. Let $\vec{v} = [2x, x + y, y]$, $\vec{u} = [2p, p + q, q]$, $x, y, p, q \in \mathbb{R}$.

(1) $\vec{v} + \vec{u} = [2x, x + y, y] + [2p, p + q, q] = [2(x + p), (x + p) + (y + q), (y + q)] \in W$

(2) $r\vec{v} = r[2x, x + y, y] = [2(rx), (rx) + (ry), ry] \in W$

Hence W is a subspace of \mathbb{R}^n

solution 2:

For any $\vec{v}, \vec{u} \in W$, any $r, s \in \mathbb{R}$. Let $\vec{v} = [2x, x + y, y]$, $\vec{u} = [2p, p + q, q]$, $x, y, p, q \in \mathbb{R}$.

$r\vec{v} + s\vec{u} = r[2x, x + y, y] + s[2p, p + q, q] = [2(rx + sp), (rx + sp) + (ry + sq), ry + sq] \in W$.

Hence W is a subspace of \mathbb{R}^n

2. (50%) (a) Find the inverse of the matrix A , if it exists, and (b) express the inverse matrix as a product of elementary matrices. $A = \begin{bmatrix} 3 & 6 \\ 3 & 8 \end{bmatrix}$

Answer: (a) $A^{-1} = \begin{bmatrix} 1.\bar{3} & -1 \\ -0.5 & 0.5 \end{bmatrix}$, (b) $A^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 1/3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$