姓名: SOLUTION 葉均承 應數一線性代數

學號: _____ **Quiz 5** 考試日期: 2020/10/29

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊! 3. 背面還有題目

1. If $T([-1,1]) = [2,1,4]$ and $T([1,1]) = [-6,3,2]$, find $T([x,y])$

Answer: $T([x,y])$= [-4x-2y, x+2y, -x+3y] .

$$\left[\begin{array}{cc|c} -1 & 1 & x \\ 1 & 1 & y \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & -x \\ 0 & 2 & x+y \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & (-x+y)/2 \\ 0 & 1 & (x+y)/2 \end{array}\right],$$

$$[x,y] = \frac{-x+y}{2}[-1,1] + \frac{x+y}{2}[1,1]$$

$$\begin{aligned} T([x,y]) &= \frac{-x+y}{2}T([-1,1]) + \frac{x+y}{2}T([1,1]) \\ &= \frac{-x+y}{2}[2,1,4] + \frac{x+y}{2}[-6,3,2] \\ &= [-4x-2y, x+2y, -x+3y] \end{aligned}$$

2. Given $A \sim H$, please answer the following questions.

$$A = \begin{bmatrix} 0 & 2 & 3 & 1 \\ -4 & 4 & 1 & 4 \\ 3 & 3 & 2 & 0 \\ -4 & 0 & 1 & 2 \end{bmatrix}, H = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) the **rank** of matrix A, is 3.

(b) a basis for the **row space** of A is [2, 0, 0, -1], [0, 2, 0, 1], [0, 0, 1, 0].

(c) a basis for the **column space** of A is $\begin{bmatrix} 0 \\ -4 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ 1 \end{bmatrix}$.

(d) a basis for the **nullspace** of A is $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}$.

(a) There's 3 pivots in matrix H.

(b) Pick the rows in **H** which contains a pivot.

(c) Pick the columns in **A** which the corresponding columns in H contains a pivot.

(d) Let $x_4 = r$. By **H**, $2x_1 - x_4 = 0, 2x_2 + x_4 = 0, x_3 = 0$. Thus $x_1 = 0.5r, x_2 = -0.5r$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = r \begin{bmatrix} 0.5 \\ -0.5 \\ 0 \\ 1 \end{bmatrix} = 0.5r \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}$$