姓名: SOLUTION 葉均承 應數一線性代數

學號: ________

Quiz 6

考試日期: 2020/11/05

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊! 3. 背面還有題目

1. Consider the set $\mathbb{R}^2$, with the addition defined by $[x, y] \oplus [a, b] = [x + a + 1, y + b]$, and with scalar multiplication defined by $r[x, y] = [rx + r - 1, ry]$.

   a. Is this set a vector space? *Hint:* Show by verifying the closed under two operations, A1-A4 and S1-S4.

   b. What is the zero vector in this vector space? *Hint:* The zero vector will NOT be the vector $[0, 0]$.

Answer: the zero vector $\vec{0} = [-1, 0]$, $-[x, y] = [-x - 2, -y]$

$[x, y] \oplus [a, b] = [x + a + 1, y + b]$ and $r[x, y] = [rx + r - 1, ry]$ are both in $\mathbb{R}^2$, hence proved the closed.

A1 $([x, y] \oplus [a, b]) \oplus [p, q] = [x + a + 1, y + b] \oplus [p, q] = [x + a + 1 + p + 1, y + b + q] = [x + a + p + 1 + 1, y + b + q] = [x, y] \oplus [a + p + 1, b + q] = [x, y] \oplus ([a, b] \oplus [p, q])$

A2 $[x, y] \oplus [a, b] = [x + a + 1, y + b] = [a + x + 1, b + y] = [a, b] \oplus [x, y]$

A3 $\vec{0} = 0[x, y] = [0x + 0 - 1, 0y] = [-1, 0]$. $\vec{0} \oplus [x, y] = [-1, 0] \oplus [x, y] = [-1 + x + 1, 0 + y] = [x, y]$

A4 $(-1)[x, y] = [-x + (-1) - 1, -y] = [-x - 2, -y]$. $[x, y] \oplus [-x - 2, -y] = [x - x - 2 + 1, y - y] = [-1, 0]$

S1 $r([x, y] \oplus [a, b)) = r([x + a + 1, y + b]) = [rx + ra + r + r - 1, ry + rb] = [rx + r - 1 + ra + r - 1 + 1, ry + rb] = [rx + r - 1, ry] \oplus [ra + r - 1, rb] = r[x, y] \oplus r[a, b]$

S2 $(r + s)[x, y] = [(r + s)x + (r + s) - 1, (r + s)y] = [rx + sx + r + s - 1, ry + sy] = [rx + r - 1 + sx + s - 1 + 1, ry + sy] = r[x, y] \oplus r[a, b]$

S3 $s(r[x, y]) = s([rx + r - 1, ry]) = [srx + sr - s + s - 1, sry] = (rs)[x, y]$

S4 $1[x, y] = [x + 1 - 1, y] = [x, y]$