姓名: SOLUTION 葉均承 應數一線性代數

學號: _____ **Quiz 13** 考試日期: 2021/06/09

此為開書考，但是禁止與其他人討論
請框出答案. 不可使用手機、計算器，禁止作弊!

1. Find all $a, b \in \mathbb{C}$ such that the following matrix is unitary diagonalizable.

$$\begin{bmatrix} a & -i \\ i & b \end{bmatrix}$$

Let

$$A = \begin{bmatrix} a & -i \\ i & b \end{bmatrix}$$

A is unitary diagonalizable if and only if A is normal.

$$AA^* = \begin{bmatrix} a & -i \\ i & b \end{bmatrix} \begin{bmatrix} \overline{a} & -i \\ i & \overline{b} \end{bmatrix} = A^*A = \begin{bmatrix} \overline{a} & -i \\ i & \overline{b} \end{bmatrix} \begin{bmatrix} a & -i \\ i & b \end{bmatrix}$$

$$\begin{bmatrix} a\overline{a}+1 & (-a-\overline{b})i \\ (\overline{a}+b)i & b\overline{b}+1 \end{bmatrix} = \begin{bmatrix} a\overline{a}+1 & (-\overline{a}-b)i \\ (a+\overline{b})i & b\overline{b}+1 \end{bmatrix}$$

A is unitary diagonalizable if and only if $(a+\overline{b}) = (\overline{a}+b)$, if and only if $a-\overline{a} = b-\overline{b}$. That is a and b has the same imaginary parts, so that $a = p+qi$ and $b = r+qi$ for some $p, q, r \in \mathbb{R}$