姓名: SOLUTION　　葉均承　　應數一線性代數

學號: _____　　**Quiz 14**　　考試日期: 2021/06/17

此為開書考，但是禁止與其他人討論
請框出答案.　不可使用手機、計算器，禁止作弊!

1. (a) Find the eigenvalues of the given Matrix J.

   (b) Give the rank and nullity of $(J-\lambda)^k$ for each eigenvalue λ of J and for every positive integer k.

   (c) Draw schemata of the strings of vectors in the standard basis arising from the Jordan blocks in J.

   (d) For each standard basis vector $\vec{e}_k$, express $J\vec{e}_k$ as a linear combination of vectors in the standard basis.

$$\begin{bmatrix} i & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

Answer:

(a) $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda_5=i$, $\lambda_6=\lambda_7=\lambda_8=2$

(b) $(J-iI)$ has rank 6 and nullity 2,
$(J-iI)^2$ has rank 4 and nullity 4,
$(J-iI)^k$ has rank 3 and nullity 5 for $k\geq 3$,
$(J-2I)$ has rank 6 and nullity 2,
$(J-2I)^k$ has rank 5 and nullity 3, for $k\geq 2$.

(c) The strings are:

$$(J-iI):\begin{cases} \vec{e}_2\to\vec{e}_1\to 0 \\ \vec{e}_5\to\vec{e}_4\to\vec{e}_3\to 0 \end{cases}$$

$$(J-2I):\begin{cases} \vec{e}_7\to\vec{e}_6\to 0 \\ \vec{e}_8\to 0 \end{cases}$$

(d) $J\vec{e}_1=i\vec{e}_1$, $J\vec{e}_2=i\vec{e}_2+\vec{e}_1$,
$J\vec{e}_3=i\vec{e}_3$, $J\vec{e}_4=i\vec{e}_4+\vec{e}_3$, $J\vec{e}_5=i\vec{e}_5+\vec{e}_4$,
$J\vec{e}_6=2\vec{e}_6$, $J\vec{e}_7=2\vec{e}_7+\vec{e}_6$,
$J\vec{e}_8=2\vec{e}_8$,