姓名: SOLUTION 葉均承 應數一線性代數

學號: ________________ **Quiz 2** 考試日期: 2020/03/11

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Find a formula for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects vectors in the line $y = mx$.

$T(\begin{bmatrix} x \\ y \end{bmatrix})= \underline{\dfrac{1}{1+m^2}\begin{bmatrix} 1-m^2x+2my \\ 2mx+(m^2-1)y \end{bmatrix}}$

$$T(\begin{bmatrix} 1 \\ m \end{bmatrix}) = \begin{bmatrix} 1 \\ m \end{bmatrix}, T(\begin{bmatrix} -m \\ 1 \end{bmatrix}) = \begin{bmatrix} m \\ -1 \end{bmatrix}$$

That is the s.m.r. of $T$ is $A = CDC^{-1}$, where

$$C = \begin{bmatrix} 1 & -m \\ m & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A = CDC^{-1} = \begin{bmatrix} 1 & -m \\ m & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\frac{1}{1+m^2}\begin{bmatrix} 1 & m \\ -m & 1 \end{bmatrix} = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$$

$$T(\begin{bmatrix} x \\ y \end{bmatrix}) = A\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2x+2my \\ 2mx+(m^2-1)y \end{bmatrix}$$

2. Let

$$A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix}$$

Find (if exists) an invertible matrix C and a diagonal matrix D such that $D = C^{-1}AC$. Also, find the eigenvalues of $A^5$.

Is $A$ diagonalizable?_____Yes!_____

If so, eigenvalues of $A^5$ are: __$3^5, (-1)^5$__, C= __$\begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix}$\_\_, and D= \_\_$\begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$__.

$$|A - \lambda I| = (7 - \lambda)(-5 - \lambda) - 8 * (-4) = (\lambda - 3)(\lambda + 1)$$

The eigenvalues for $A$ are $3, -1$, and the eigenvalues for $A^5$ are $3^5, (-1)^5$.

$\boxed{\lambda = 3}$,

$$A - 3I = \begin{bmatrix} 4 & 8 \\ -4 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \Rightarrow \vec{v} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$\boxed{\lambda = -1}$,

$$A + I = \begin{bmatrix} 8 & 8 \\ -4 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow \vec{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix}, D = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$$