姓名: SOLUTION　　葉均承　　應數一線性代數

學號: _____　　**Quiz 3**　　考試日期: 2020/03/18

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Solve the given system.

$$\begin{cases} x_1' = -2x_1 - x_3 \\ x_2' = 2x_2 \\ x_3' = 3x_1 - 2x_3 \end{cases}$$

Answer: SOLUTION

$$A = \begin{bmatrix} -2 & 0 & -1 \\ 0 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}, C = \begin{bmatrix} -1 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 3 & 0 \end{bmatrix}, D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

Such that $AC = CD$, and $\vec{x}' = A\vec{x}$

$$\text{Solve } \vec{y}' = D\vec{y}, \text{ i.e. } \begin{cases} y_1' = -y_1 \\ y_2' = y_2 \\ y_3' = 2y_3 \end{cases} \Rightarrow \begin{cases} y_1 = k_1 e^{-t} \\ y_2 = k_2 e^{t} \\ y_3 = k_3 e^{2t} \end{cases}$$

$$\vec{x} = C\vec{y} \Rightarrow \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -k_1 e^{-t} - k_2 e^{t} \\ k_3 e^{2t} \\ k_1 e^{-t} + 3k_2 e^{t} \end{bmatrix}$$