姓名: SOLUTION 葉均承 應數一線性代數

學號: ________________ **Quiz 7** 考試日期: 2021/04/15

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Find the projection matrix for the plane $x + y + 2z = 0$ and then find the projection of $[1, 4, 1]$ on the plane.

Answer: $\vec{b}_W = P\vec{b} = \frac{1}{6}[-1, 17, -8]$

Let W is the plane $x + y + 2z = 0$. Notice that $[0, 0, 0]$ in W.
I will provides **3 different ways** to solve this question.

Solution 1 (Method from 6.4 example 3)
Pick $\vec{a}_1 = [-2, 0, 1]^T, \vec{a}_2 = [0, -2, 1]^T$ such that $W = sp(\vec{a}_1, \vec{a}_2)$.

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -2 \\ 1 & 1 \end{bmatrix}, (A^TA)^{-1} = \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}^{-1} = \frac{1}{24}\begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix}$$

The projection matrix P is

$$P = A(A^TA)^{-1}A^T = \frac{1}{24}\begin{bmatrix} -2 & 0 \\ 0 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 0 & -2 & 1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 5 & -1 & -2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}$$

$$\vec{b}_W = P\vec{b} = \frac{1}{6}\begin{bmatrix} 5 & -1 & -2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} -1 \\ 17 \\ -8 \end{bmatrix}$$

Solution 2 (Method from 6.2 example 1, example 2)
The normal vector of W is $\vec{n} = [1, 1, 2]$, and pick another $\vec{v}_1 = [-2, 0, 1]$. Then

$$\vec{v}_2 = [1, 1, 2] \times [-2, 0, 1] = [1, -5, 2].$$

Hence $W = sp(\vec{v}_1, \vec{v}_2)$, and that is an orthogonal basis for W.

$$\begin{aligned} \vec{b}_W &= \frac{\vec{b}\cdot\vec{v}_1}{\vec{v}_1\cdot\vec{v}_1}\vec{v}_1 + \frac{\vec{b}\cdot\vec{v}_2}{\vec{v}_2\cdot\vec{v}_2}\vec{v}_2 \\ &= \frac{-1}{5}[-2, 0, 1] + \frac{-17}{30}[1, -5, 2] = \frac{1}{6}[-1, 17, -8] \end{aligned}$$

Solution 3 (Method from 6.1 example 3)

Let $\vec{b} = [1,4,1]^T$. Pick $\vec{a}_1 = [-2,0,1]^T, \vec{a}_2 = [0,-2,1]^T$ such that $W = sp(\vec{a}_1, \vec{a}_2)$. The $W^\perp = sp(\vec{n}) = sp([1,1,2]^T)$.

$$\left[\vec{a}_1 \quad \vec{a}_2 \quad \vec{n} \,\middle|\, \vec{b}\right] = \left[\begin{array}{ccc|c} -2 & 0 & 1 & 1 \\ 0 & -2 & 1 & 4 \\ 1 & 1 & 2 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/12 \\ 0 & 1 & 0 & -17/12 \\ 0 & 0 & 1 & 14/12 \end{array}\right]$$

$$\vec{b} = \frac{1}{12}\vec{a}_1 + \frac{-17}{12}\vec{a}_2 + \frac{14}{12}\vec{n}$$

$$\vec{b}_W = \frac{1}{12}\vec{a}_1 + \frac{-17}{12}\vec{a}_2 = \frac{1}{12}\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + \frac{-17}{12}\begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} -1 \\ 17 \\ -8 \end{bmatrix}$$