姓名: SOLUTION 葉均承 應數一線性代數

學號: ________________ **Quiz 8** 考試日期: 2021/05/06

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Find the change-of-coordinates matrix from B to B' and from B' to B, indicate which is which.

$$B = (\sin^2(x), \cos^2(x)) \text{ and } B' = (1, \cos(2x))$$

Answer: $C_{B,B'}=$ $\frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$, $C_{B',B}=$ $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

It is obviously that

$$\begin{cases} \sin^2(x) + \cos^2(x) & = & 1 \\ -\sin^2(x) + \cos^2(x) & = & \cos(2x) \end{cases} \Rightarrow \begin{cases} (1)_B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ (\cos(2x))_B = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \end{cases}$$

$$C_{B',B} = \begin{bmatrix} | & | & & | \\ (\vec{b}'_1)_B & (\vec{b}'_2)_B & \dots & (\vec{b}'_n)_B \\ | & | & & | \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$C_{B,B'} = C_{B',B}^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

2. Let V be a vector space with ordered bases B and B'. If

$$C_{B,B'} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

and $\vec{v} = \vec{b}_1 - 3\vec{b}_2$, find the coordinate vector $\vec{v}_{B'}$.

Answer: $\vec{v}_{B'} = \underline{\begin{bmatrix} -5 \\ -3 \end{bmatrix}}$

$$\vec{v} = \vec{b}_1 - 3\vec{b}_2 \Rightarrow \vec{v}_B = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$\because \vec{v}_{B'} = C_{B,B'}\vec{v}_B$$

$$\therefore \vec{v}_{B'} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \end{bmatrix}$$