1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Find the matrix representations $R_{B,B}$, $R_{B',B'}$ and an invertible C such that $R_{B',B'} = C^{-1}R_{B,B}C$ for the given linear transformation T.
$T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T([x, y, z]) = [5x, 5y, 5z]$; $B = ([1, 1, 1], [1, 1, 0], [1, 0, 0])$, $B' = E$.
$C_{B,B'}= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $C_{B',B}= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$, $R_{B',B'}= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ and $R_{B,B}= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$.
Is C=$C_{B,B'}$ or $C_{B',B}$? $\underline{C_{B'B}}$.

By $T([x, y, z]) = [5x, 5y, 5z]$, we have

$$R_{B',B'} = R'_B = R_E = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$C_{B,B'} = M_{B'}^{-1}M_B = M_E^{-1}M_B = I^{-1}M_B = M_B$.

$$C_{B,B'} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$C = C_{B',B} = C_{B,B'}^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

Since

$$T([1, 1, 1]) = [5, 5, 5],\ T([1, 1, 0]) = [5, 5, 0],\ T([1, 0, 0]) = [5, 0, 0],$$

$$\because \left[M_B \mid M_{T(B)}\right] \sim [I \mid R_B]$$

$$\therefore \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 5 & 5 & 5 \\ 1 & 1 & 0 & 5 & 5 & 0 \\ 1 & 0 & 0 & 5 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & 0 & 0 \\ 0 & 1 & 0 & 0 & 5 & 0 \\ 0 & 0 & 1 & 0 & 0 & 5 \end{array}\right]$$

Thus

$$R_{B,B} = R_B = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

多給一個示範題在背面

2. Find the matrix representations $R_{B,B}$, $R_{B',B'}$ and an invertible C such that $R_{B',B'} = C^{-1}R_{B,B}C$ for the given linear transformation T.
$T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T([x,y,z]) = [5x, 2y, 3z]$; $B = ([1,1,1],[1,1,0],[1,0,0])$, $B' = E$.
$C_{B,B'}=$ $\begin{bmatrix}1&1&1\\1&1&0\\1&0&0\end{bmatrix}$, $C_{B',B}=$ $\begin{bmatrix}0&0&1\\0&1&-1\\1&-1&0\end{bmatrix}$, $R_{B',B'}=$ $\begin{bmatrix}5&0&0\\0&2&0\\0&0&3\end{bmatrix}$ and $R_{B,B}=$ $\begin{bmatrix}3&0&0\\-1&2&0\\3&3&5\end{bmatrix}$.
Is C=$C_{B,B'}$ or $C_{B',B}$? $C_{B'B}$.

By $T([x,y,z]) = [5x, 2y, 3z]$, we have

$$R_{B',B'} = R'_B = R_E = \begin{bmatrix}5&0&0\\0&2&0\\0&0&3\end{bmatrix}$$

$C_{B,B'} = M_{B'}^{-1}M_B = M_E^{-1}M_B = I^{-1}M_B = M_B$.

$$C_{B,B'} = \begin{bmatrix}1&1&1\\1&1&0\\1&0&0\end{bmatrix}$$

$$C = C_{B',B} = C_{B,B'}^{-1} = \begin{bmatrix}1&1&1\\1&1&0\\1&0&0\end{bmatrix}^{-1} = \begin{bmatrix}0&0&1\\0&1&-1\\1&-1&0\end{bmatrix}$$

Since

$$T([1,1,1]) = [5,2,3],\ T([1,1,0]) = [5,2,0],\ T([1,0,0]) = [5,0,0],$$

$$\because \left[M_B \mid M_{T(B)}\right] \sim \left[I \mid R_B\right]$$

$$\therefore \left[\begin{array}{ccc|ccc}1&1&1&5&5&5\\1&1&0&2&2&0\\1&0&0&3&0&0\end{array}\right] \sim \left[\begin{array}{ccc|ccc}1&0&0&3&0&0\\0&1&0&-1&2&0\\0&0&1&3&3&5\end{array}\right]$$

Thus

$$R_{B,B} = R_B = \begin{bmatrix}3&0&0\\-1&2&0\\3&3&5\end{bmatrix}$$