姓名: SOLUTION 葉均承 應數一線性代數

學號: _____ **Quiz 10** 考試日期: 2021/12/08

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!
3. 請自備白紙書寫，作答完畢請拍照上傳 Googld Classroom
4. 照片請清晰並轉正

1. Let $T : F \to \mathbb{R}$, where F be the set contains all function from $\mathbb{R}$ to $\mathbb{R}$. $T(f(x)) = f(3)$. Determine (a) whether T is an linear transformation, (b) if so, whether T is invertible.

   Answer: (a) T is an linear transformation: yes. (b) T is invertible: no

   (a) For any $f, g \in F$, any $r \in \mathbb{R}$:

   1. $T((f+g)(x)) = (f+g)(3) = f(3) + g(3) = T(f(x)) + T(g(x))$.
   2. $T((rf)(x)) = (rf)(3) = rf(3) = rT(f(x))$.

   Hence, T is an linear transformation.

   (b) T is not an one-to-one linear transformation since $T(x-3) = 0 = T(x^2-9)$. Hence T is not invertible.