姓名: SOLUTION　　　　　　　　　　　　　　　　　　　　　　葉均承　　應數一線性代數

學號: ______________

Quiz 11

考試日期: 2021/12/16

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!
3. 作答完畢請拍照上傳 Googld Classroom
4. 照片請清晰並轉正

1. Let V and V' be vector spaces with ordered basis $B = (\vec{b}_1, \vec{b}_2, \vec{b}_3, \vec{b}_4)$ and $B' = (\vec{b}'_1, \vec{b}'_2, \vec{b}'_3)$, respectively. $T : V \to V'$, be the linear transformation having matrix A as matrix representation relative to B, B'. Find $T(\vec{v})$ and $ker(T)$.

$$A = \begin{bmatrix} 4 & 2 & 0 & 2 \\ 1 & 2 & 6 & 1 \\ 0 & -6 & -20 & 0 \end{bmatrix}, \ \vec{v} = \vec{b}_1 - \vec{b}_2 + \vec{b}_3 + 2\vec{b}_4$$

Answer: (a) $T(\vec{v})=$ $6\ \vec{b}'_1 + 7\ \vec{b}'_2 - 14\ \vec{b}'_3$. (b) the $ker(T) =$ $\left\{ r\left(\frac{-4}{3}\vec{b}_1 + \frac{5}{3}\vec{b}_2 - \frac{1}{2}\vec{b}_3 + \vec{b}_4\right) \middle|\ r \in \mathbb{R} \right\}$.

$$\vec{v}_B = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \ A\vec{v}_B = \begin{bmatrix} 6 \\ 7 \\ -14 \end{bmatrix}$$

Hence $T(\vec{v}) = 6\ \vec{b}'_1 + 7\ \vec{b}'_2 - 14\ \vec{b}'_3$

$$rref(A) = \begin{bmatrix} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 0 & -5/3 \\ 0 & 0 & 1 & 1/2 \end{bmatrix}$$

Hence the kernel of A is

$$\left\{ r \begin{bmatrix} -4/3 \\ 5/3 \\ -1/2 \\ 1 \end{bmatrix} \middle|\ r \in \mathbb{R} \right\}.$$

Therefore, the kernel of T is

$$\left\{ r\left(\frac{-4}{3}\vec{b}_1 + \frac{5}{3}\vec{b}_2 - \frac{1}{2}\vec{b}_3 + \vec{b}_4\right) \middle|\ r \in \mathbb{R} \right\}$$