Quiz 2

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!
3. 請自備白紙書寫，作答完畢請拍照上傳 Googld Classroom
4. 照片請清晰並轉正

1. Find all possible scalar r such that the matrix A commutes with matrix B.

$$A = \begin{bmatrix} r & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Answer: $\underline{r = 3}$.

$$AB = \begin{bmatrix} r & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} r & 0 & r \\ 0 & 1 & 0 \\ 3 & 0 & 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} r & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} r & 0 & 3 \\ 0 & 1 & 0 \\ r & 0 & 3 \end{bmatrix}$$

Therefore,$AB = BA$ if and only if $r = 3$.

2. For vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^n$, prove that $\vec{v} - \vec{w}$ and $\vec{v} + \vec{w}$ are perpendicular if and only if $\|\vec{v}\| = \|\vec{w}\|$

Solution:

$\vec{v} - \vec{w}$ and $\vec{v} + \vec{w}$ are perpendicular means $(\vec{v} - \vec{w}) \cdot (\vec{v} + \vec{w}) = (\vec{v} + \vec{w}) \cdot (\vec{v} - \vec{w}) = 0$. Then

$$0 = (\vec{v} - \vec{w}) \cdot (\vec{v} + \vec{w}) = \vec{v} \cdot \vec{v} + \vec{v} \cdot \vec{w} - \vec{w} \cdot \vec{v} - \vec{w} \cdot \vec{w} = \vec{v} \cdot \vec{v} - \vec{w} \cdot \vec{w} = \|\vec{v}\|^2 - \|\vec{w}\|^2$$

Since $\|\vec{v}\|$ and $\|\vec{w}\|$ are both non-negative, we know $\|\vec{v}\|^2 = \|\vec{w}\|^2$ if and only if $\|\vec{v}\| = \|\vec{w}\|$.