1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!
3. 請自備白紙書寫，作答完畢請拍照上傳 Googld Classroom
4. 照片請清晰並轉正

1. Determine whether the given subset is a subspace of $\mathbb{R}^3$. Please give reasons to support your answer.

$$\{[2y + z, \; y, \; z] \mid y, \; z \in \mathbb{R}\}$$

Circle the answer: ($\boxed{Yes}$ / NO), and write your reason below.

Answer:

$W = \{[2y + z, \; y, \; z] \mid y, \; z \in \mathbb{R}\}$ is nonempty since $[0, \; 0, \; 0] \in W$.

1. Let $[2y_1 + z_1, \; y_1, \; z_1]$ and $[2y_2 + z_2, \; y_2, \; z_2]$ be in W.

$[2y_1 + z_1, \; y_1, \; z_1] + [2y_2 + z_2, \; y_2, \; z_2] = [2y_1 + z_1 + 2y_2 + z_2, \; y_1 + y_2, \; z_1 + z_2]$
$= [2(y_1 + y_2) + (z_1 + z_2), \; (y_1 + y_2), \; (z_1 + z_2)] \in W$

2. Let $[2y_1 + z_1, \; y_1, \; z_1] \in W$ and $r \in \mathbb{R}$.

$r[2y_1 + z_1, \; y_1, \; z_1] = [r(2y_1 + z_1), \; ry_1, \; rz_1] = [(r2y_1) + (rz_1), \; (ry_1), \; (rz_1)] \in W$

Thus W is nonempty and closed under addition and scalar multiplication, so it is a subspace of $\mathbb{R}^2$.

2. (a) Find the inverse of the matrix A, if it exists, and (b) express the inverse matrix as a product of elementary matrices. $A = \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}$

Answer: (a) $A^{-1} = \begin{bmatrix} 2 & -0.75 \\ -1 & 0.5 \end{bmatrix}$, (b) $A^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$