姓名: SOLUTION 葉均承 應數一線性代數

學號: _____ **Quiz 5** 考試日期: 2021/10/21

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!
3. 請自備白紙書寫，作答完畢請拍照上傳 Googld Classroom
4. 照片請清晰並轉正

1. Solve the given linear system and express the solution set.

$$\begin{cases} 2x_1 - x_2 + 4x_3 - 2x_4 = 0 \\ x_1 - 2x_2 + 3x_3 - 2x_4 = 0 \\ 2x_2 + x_3 - x_4 = -7 \\ x_1 - 2x_2 + x_4 = 9 \end{cases}$$

Answer: the solution set is $\left\{ \begin{bmatrix} 5 \\ -2 \\ -3 \\ 0 \end{bmatrix} + r \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \middle| \, r \in \mathbb{R} \right\}$,

Answer:

Let $[A|\vec{b}] = \left[\begin{array}{cccc|c} 2 & -1 & 4 & -2 & 0 \\ 1 & -2 & 3 & -2 & 0 \\ 0 & 2 & 1 & -1 & -7 \\ 1 & -2 & 0 & 1 & 9 \end{array}\right]$, and $[H|\vec{c}] = rref([A|\vec{b}]) = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

Assume $x_4 = 0$, plug into $[H|\vec{c}]$. We have $\begin{cases} x_1 = 5 \\ x_2 = -2 \\ x_3 = -3 \end{cases}$.

Hence, We have a particular solution $\begin{bmatrix} 5 \\ -2 \\ -3 \\ 0 \end{bmatrix}$.

The solution set is $\left\{ \begin{bmatrix} 5 \\ -2 \\ -3 \\ 0 \end{bmatrix} + r \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \middle| \, r \in \mathbb{R} \right\}$