

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!
3. 請自備白紙書寫，作答完畢請拍照上傳 Googld Classroom
4. 照片請清晰並轉正

1. Let the subspace $W = \text{sp}([1, -3, 2], [2, -5, 3], [-2, 3, -1])$ in \mathbb{R}^3 .

(a) Find $\dim(W)$.

(b) Find a basis for W .

(c) Is $\dim(W) = 3$? If not, enlarge the basis you get in (b) to be a basis for \mathbb{R}^3 .

Answer:

$$\text{Let } A = \begin{bmatrix} 1 & 2 & -2 & 1 & 0 & 0 \\ -3 & -5 & 3 & 0 & 1 & 0 \\ 2 & 3 & -1 & 0 & 0 & 1 \end{bmatrix}, \text{ and } H = \text{rref}(A) = \begin{bmatrix} 1 & 0 & 4 & 0 & 3 & 5 \\ 0 & 1 & -3 & 0 & -2 & -3 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Since the pivots are in the 1st, 2nd and 4th column of H , we have:

1. The $\dim(W) = 2$.

2. A basis for W is $\{[1, -3, 2], [2, -5, 3]\}$.

3. A requested basis for \mathbb{R}^3 is $\{[1, -3, 2], [2, -5, 3], [1, 0, 0]\}$.