姓名: SOLUTION 葉均承 應數一線性代數

學號: _____ **Quiz 7** 考試日期: 2021/11/04

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!
3. 請自備白紙書寫，作答完畢請拍照上傳 Googld Classroom
4. 照片請清晰並轉正

1. Is $T([x,y]) = [x-y, 3x-2y, y+1]$ a linear transformation of $\mathbb{R}^2$ to $\mathbb{R}^3$? Why or why not?

$$\begin{aligned} T([x,y]+[a,b]) &= T([x+a, y+b]) \\ &= [x+a-(y+b), 3(x+a)-2(y+b), (y+b)+1] \\ &= [x+a-y-b, 3x+3a-2y-2b, y+b+1] \end{aligned}$$

$$\begin{aligned} T(x,y)+T([a,b]) &= [x-y, 3x-2y, y+1]+[a-b, 3a-2b, b+1] \\ &= [x-y+a-b, 3x-2y+3a-2b, y+b+2] \end{aligned}$$

Since $T(x,y)+T([a,b]) \neq T([x,y]+[a,b])$, T is NOT a a linear transformation.

2. Given $A \sim H$, please answer the following questions.

$$A = \begin{bmatrix} 2 & -3 & 0 & 1 & 4 \\ 1 & 4 & -6 & 3 & -2 \\ 0 & 11 & 2 & 5 & -8 \\ -4 & 17 & -12 & 3 & -16 \end{bmatrix}, H = \begin{bmatrix} 11 & 0 & 0 & 13 & 10 \\ 0 & 11 & 0 & 5 & -8 \\ 0 & 0 & 11 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) the **rank** of matrix A, is _____3_____.

(b) a basis for the **row space** of A is _____[11, 0, 0, 13, 10], [0, 11, 0, 5, -8], [0, 0, 11, 0, 0]_____.

(c) a basis for the **column space** of A is _____ $\begin{bmatrix} 2 \\ 1 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 11 \\ 17 \end{bmatrix}, \begin{bmatrix} 0 \\ -6 \\ 2 \\ -12 \end{bmatrix},$ _____.

(d) a basis for the **nullspace** of A is _____ $\begin{bmatrix} -13/11 \\ -5/11 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -10/11 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ _____.

(a) There's 3 pivots in matrix H.

(b) Pick the rows in **H** which contains a pivot.

(c) Pick the columns in **A** which the corresponding columns in H contains a pivot.

(d) Let $x_4 = r, x_5 = s$. By **H**, $11x_1 + 13x_4 + 10x_5 = 0, 11x_2 + 5x_4 - x_5 = 0, 11x_3 = 0$. Thus $x_1 = \frac{-13}{11}r - \frac{10}{11}s, x_2 = \frac{-5}{11}r + s, x_3 = 0$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = r \begin{bmatrix} -13/11 \\ -5/11 \\ 0 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -10/11 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$