Quiz 10

不可使用手機、計算器，禁止作弊!

1. Express z/w in the form $a + bi$, where $a, b \in \mathbb{R}$, if

$$z = 1 + 2i,\ w = 4 - 3i$$

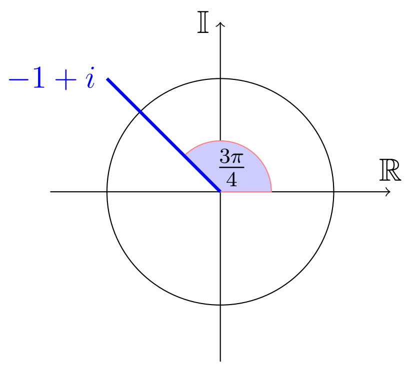
Answer: $z/w =$ $\frac{-2+11i}{25}$

$$1/w = \frac{\overline{w}}{|w|^2} = \frac{4+3i}{16+9}$$

$$z/w = \frac{z\overline{w}}{|w|^2} = \frac{(1+2i)(4+3i)}{16+9} = \frac{-2+11i}{25}$$

2. Find the modulus and principal argument of $(i - 1)$.

Answer: the modulus/ norm/ length is = $\sqrt{2}$, the principal argument is $\frac{3\pi}{4}$



$$|i - 1| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$i - 1 = \sqrt{2}\left(\frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$$

3. Find the three cube roots of $(i - 1)$.

Answer: w_0, w_1, w_2.

$$i - 1 = \sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$$

$$w_k = \sqrt[6]{2}\left(\cos\left(\frac{\pi}{4} + \frac{2k\pi}{3}\right) + i\sin\left(\frac{\pi}{4} + \frac{2k\pi}{3}\right)\right)$$