姓名: SOLUTION 葉均承 應數一線性代數

學號: ______________

Quiz 11

考試日期: 2022/05/26

不可使用手機、計算器，禁止作弊!

1. Solve the linear system

$$\begin{cases} (1+i)z_1 & +(1+2i)z_2 & = & 3 \\ (2-i)z_1 & +(1-i)z_2 & = & 1 \end{cases}$$

Answer: $(z_1, z_2) = \left(\frac{11+16i}{13}, \frac{-2-23i}{13}\right)$

The original problem can be rewrithed as the following

$$\begin{bmatrix} 1+i & 1+2i \\ 2-i & 1-i \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Hence,

$$\begin{aligned} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} &= \begin{bmatrix} 1+i & 1+2i \\ 2-i & 1-i \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 1 \end{bmatrix} \\ &= \frac{1}{(1+i)(1-i)-(1+2i)(2-i)} \begin{bmatrix} 1-i & -(1+2i) \\ -(2-i) & 1+i \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} \\ &= \frac{1}{-2-3i} \begin{bmatrix} 2-5i \\ -5+4i \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 11+16i \\ -2-23i \end{bmatrix} \end{aligned}$$

2. find $< [1+i, 1-2i, 3-i], [1-2i, 1+i, i] >$.

Answer: $< [1+i, 1-2i, 3-i], [1-2i, 1+i, i] >=$ $-3+3i$

$$< [1+i, 1-2i, 3-i], [1-2i, 1+i, i] >= (\overline{1+i})(1-2i) + (\overline{1-2i})(1+i) + (\overline{3-i})(i)$$

$$= (1-i)(1-2i) + (1+2i)(1+i) + (3+i)(i) = (-1-3i) + (-1+3i) + (-1+3i) = -3+3i$$