姓名: SOLUTION 葉均承 應數一線性代數

學號: ______________

Quiz 12

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不可使用手機、計算器，禁止作弊!

1. Find a vector perpendicular to both $[1+i, 2-i, i], [1-i, 1, 1-i]$

   Answer: $[1+4i, -1-i, -4i]$

$$\begin{bmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ 1+i & 2-i & i \\ 1-i & 1 & 1-i \end{bmatrix} = [1+4i, -1-i, -4i]$$

2. Find an invertible matrix C and a diagonal matrix D such that $D = C^{-1}AC$, where

$$A = \begin{bmatrix} 2-i & 0 & 2i \\ 0 & 1 & 0 \\ -i & 0 & 2+i \end{bmatrix}$$

   Answer: $C = \begin{bmatrix} 1-i & 0 & 1+i \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$,

$$|A-\lambda I| = \begin{vmatrix} 2-i-\lambda & 0 & 2i \\ 0 & 1-\lambda & 0 \\ -i & 0 & 2+i-\lambda \end{vmatrix} = (1-\lambda)[(2-\lambda)^2 + 1 + 2i^2] = (1-\lambda)^2(3-\lambda)$$

$\boxed{\lambda = 1}$

$$rref(A-I) = \begin{bmatrix} 1 & 0 & -1+i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \vec{v} = r\begin{bmatrix} 1-i \\ 0 \\ 1 \end{bmatrix} + s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\boxed{\lambda = 3}$

$$rref(A-3I) = \begin{bmatrix} 1 & 0 & -1-i \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \vec{v} = \begin{bmatrix} 1+i \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore \lambda_1 = \lambda_2 = 1,\ \vec{v}_1 = \begin{bmatrix} 1-i \\ 0 \\ 1 \end{bmatrix},\ \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ \lambda_3 = 3,\ \vec{v}_3 = \begin{bmatrix} 1+i \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} | & | & | \\ \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 1-i & 0 & 1+i \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix},\ D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

P.S. Since $AA^* \neq A^*A$, A is not a normal matrix. Therefore, A can not be unitary diagonalizable.