姓名: SOLUTION　　葉均承　　應數一線性代數

學號: _____　　**Quiz 14**　　考試日期: 2022/06/16

不可使用手機、計算器，禁止作弊!

1. Find a Jordan canonical form for A, where $(A-3I)$ has nullity 2, $(A-3I)^2$ has nullity 3, $(A-3I)^3$ has nullity 4, $(A-3I)^k$ has nullity 5 for $k \geq 4$; $(A+I)^K$ has nullity 1 for $k \geq 1$; $(A-2I)$ has nullity 2, $(A-2I)^2$ has nullity 4 for $k \geq 2$.

   Answer:

   $(A-3I)$ has nullity 2,
   $(A-3I)^2$ has nullity 3,
   $(A-3I)^3$ has nullity 4,
   $(A-3I)^k$ has nullity 5 for $k \geq 4$
   $\Rightarrow$ $(A-3I)$: $\vec{e}_1 \to 0$, $\vec{e}_5 \to \vec{e}_4 \to \vec{e}_3 \to \vec{e}_2 \to 0$

   $(A+I)^K$ has nullity 1 for $k \geq 1$ $\Rightarrow$ $(A+I)$: $\vec{e}_6 \to 0$

   $(A-2I)$ has nullity 2,
   $(A-2I)^2$ has nullity 4 for $k \geq 2$
   $\Rightarrow$ $(A-2I)$: $\vec{e}_8 \to \vec{e}_7 \to 0$, $\vec{e}_9 \to \vec{e}_{10} \to 0$

$$\begin{bmatrix} \boxed{3} & & & \\ & \boxed{\begin{matrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{matrix}} & & \\ & & \boxed{-1} & \\ & & & \boxed{\begin{matrix} 2 & 1 \\ 0 & 2 \end{matrix}} \end{bmatrix}$$

2. Mark all the matrix if it is a Jordan Canonical form and boxed all the Jordan blocks in it.

$\boxed{Yes}$ / No　　(a) $\begin{bmatrix} \boxed{\begin{matrix} 3 & 1 \\ 0 & 3 \end{matrix}} & 0 & 0 \\ 0 & 0 & \boxed{2} & 0 \\ 0 & 0 & 0 & \boxed{2} \end{bmatrix}$

$\boxed{Yes}$ / No (b) $\begin{bmatrix} \boxed{1} & 1 & 0 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & \boxed{1} \end{bmatrix}$

$\boxed{Yes}$ / No (c) $\begin{bmatrix} \boxed{0} & 1 & 0 & 0 \\ 0 & \boxed{0} & 1 & 0 \\ 0 & 0 & \boxed{0} & 0 \\ 0 & 0 & 0 & \boxed{0} \end{bmatrix}$

$\boxed{Yes}$ / No (d) $\begin{bmatrix} \boxed{1} & 0 & 0 & 0 \\ 0 & \boxed{1} & 0 & 0 \\ 0 & 0 & \boxed{3} & 0 \\ 0 & 0 & 0 & \boxed{3} \end{bmatrix}$

Yes / $\boxed{No}$ (e) $\begin{bmatrix} i & 1 & 0 & 0 \\ 0 & i & 1 & 0 \\ 0 & 0 & i & 1 \\ 0 & 0 & 0 & \cancel{0} \end{bmatrix}$