姓名: SOLUTION 葉均承 應數一線性代數

學號: _____ **Quiz 1** 考試日期: 2022/03/03

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Find the characteristic polynomial, the eigenvalues, and the corresponding eigenvectors of the following matrix.

$$A = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}$$

Answer: (a) the characteristic polynomial: $(-1-\lambda)(2-\lambda)(3-\lambda)$.

(b) the eigenvalues and the corresponding eigenvectors: $(2, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}), (3, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}), (-1, \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix})$

Answer:

(a)

$$\begin{aligned} p_A(\lambda) = A - \lambda I &= \begin{vmatrix} 2-\lambda & 4 & 2 \\ 0 & 1-\lambda & -2 \\ 0 & -2 & 1-\lambda \end{vmatrix} \\ &= (2-\lambda)(1-\lambda)(1-\lambda) - (-2)(-2)(2-\lambda) \\ &= (2-\lambda)[1-2\lambda+\lambda^2-4] \\ &= (2-\lambda)(-1-\lambda)(3-\lambda) \end{aligned}$$

p.s. 請特別注意，上面是用行列式值的符號，而不是矩陣符號。

(b)

$\boxed{\lambda = -1}$

$$A-(-1)I = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -2 \\ 0 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{eigenvector is } \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

$\boxed{\lambda = 2}$

$$A - 2I = \begin{bmatrix} 0 & 4 & 2 \\ 0 & -1 & -2 \\ 0 & -2 & -1 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{eigenvector is } \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{\lambda = -1}$

$$A - 3I = \begin{bmatrix} -1 & 4 & 2 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{eigenvector is } \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$