姓名: SOLUTION 葉均承 應數一線性代數

學號: _____ **Quiz 2** 考試日期: 2022/03/10

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Circle each of the following True or False. If it is False, please explain why.

   (a) **True** False If $\vec{v}$ is an eigenvector of a matrix A, then $\vec{v}$ is an eigenvector of $A + cI$ for all scalars c.

   If $A\vec{v} = \lambda\vec{v}$, then $(A + cI)\vec{v} = (\lambda + c)\vec{v}$. Matrix $A + cI$ has eigenvector $\vec{v}$ corresponding to eigenvalue $(\lambda + c)$.

   (b) True **False** If λ is an eigenvalue of a matrix A, then λ is an eigenvalue of $A + cI$ for all scalars c.

   Pick $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $c = 2$. The only eigenvalue of A is $\lambda = 1$, while the only eigenvalue of matrix $A + 2I = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ is 3.

   (c) True **False** An $n \times n$ matrix is diagonalizable if and only if it has n distinct eigenvalues.

   By **Theorem 5.3** from the textbook: any $n \times n$ matrix which has n distinct eigenvalues will be diagonalizable. However, the reverse statement is not always true. For example, a diagonalizable matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ has 1 as its only eigenvalue.

2. Let

$$A = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -2 & 1 \\ 2 & 0 & -1 \end{bmatrix}$$

   Find (if exists) an invertible matrix C and a diagonal matrix D such that $D = C^{-1}AC$. Also, find the eigenvalues of A^5.

   Is A diagonalizable? No!. If so, C= _____, and D=_____.

   $p(\lambda) = (\lambda - 1)(\lambda + 2)^2$, then $\lambda = 1, (-2), (-2)$.

$$rref(A - (-2)I) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

   For eigenvalue (-2), its algebraic multiplicity is 2, but its geometric multiplicity is 1. Since $2 \neq 1$, A is NOT diagonalizable.