

不可使用手機、計算器，禁止作弊!
背面還有題目

1. Find an orthonormal basis for the plane $3x + y - 2z = 0$ in \mathbb{R}^3

Answer: $\{\frac{1}{\sqrt{5}}[0, 2, 1], \frac{1}{\sqrt{70}}[5, -3, 6]\}$.

The normal vector \vec{n} of the plane is $[3, 1, -2]$

Pick two points $(0, 0, 0)$ and $(0, 2, 1)$ in the plane, therefore, the vector $\vec{a} = [0, 2, 1]$ in the plane.

$$\text{Let } \vec{b} = \vec{a} \times \vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -2 \\ 0 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} \vec{i} - \begin{vmatrix} 3 & -2 \\ 0 & 1 \end{vmatrix} \vec{j} + \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} \vec{k} = [5, -3, 6]$$

$\{\vec{a}, \vec{b}\} = \{[0, 2, 1], [5, -3, 6]\}$ is an orthogonal basis for the plane.

$\{\frac{1}{\sqrt{5}}[0, 2, 1], \frac{1}{\sqrt{70}}[5, -3, 6]\}$ is an orthonormal basis for the plane.