不可使用手機、計算器，禁止作弊!
背面還有題目

1. Given $T : \mathbb{R}^3 \to \mathbb{R}^3$, where $T([x,y,z]) = \frac{1}{3}[2x-2y+z, x+2y+2z, 2x+y-2z]$.

   (1) Determine whether the linear transformation T is orthogonal.

   Answer: T is orthogonal.

   (2) Find the angle between $T([1,0,1])$ and $T([0,1,0])$

   Answer: $\frac{\pi}{2}$.

$$T([x,y,z])^T = \frac{1}{3}\begin{bmatrix} 2x-2y+z \\ x+2y+2z \\ 2x+y-2z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Therefore, the standard matrix representation of T is

$$A = \frac{1}{3}\begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{bmatrix}$$

Since $A^T A = I$, T is orthogonal!

By **Theorem of Orthogonal Linear Transformation**:
The angle between $T(\vec{x})$ and $T(\vec{y})$ equals the angle between $\vec{x}$ and $\vec{y}$.

Let the $\vec{x} = [1,0,1]$, $\vec{y} = [0,1,0]$. Since $\vec{x}$ and $\vec{y}$ are obviously perpendicular, $T(\vec{x})$ and $T(\vec{y})$ are also perpendicular. Hence, the angle between $T(\vec{x})$ and $T(\vec{y})$ is $\frac{\pi}{2}$.