姓名: SOLUTION 葉均承 應數一線性代數

學號: _____ **Quiz 7** 考試日期: 2022/04/13

不可使用手機、計算器，禁止作弊!

1. Find the projection matrix for the plane $x+2y-z=0$ and then find the projection of $[2,1,3]$ on the plane.

Answer: $P=\frac{1}{6}\begin{bmatrix}5&-2&1\\-2&2&2\\1&2&5\end{bmatrix}$, $\vec{b}_W=\frac{1}{6}\begin{bmatrix}11\\4\\19\end{bmatrix}$

(Method from 6.4 example 3)

Pick $\vec{a}_1=[-2,1,0]^T, \vec{a}_2=[0,1,2]^T$ such that $W=sp(\vec{a}_1,\vec{a}_2)$.

$$A=\begin{bmatrix}-2&0\\1&1\\0&2\end{bmatrix}, (A^TA)^{-1}=\begin{bmatrix}5&1\\1&5\end{bmatrix}^{-1}=\frac{1}{24}\begin{bmatrix}5&-1\\-1&5\end{bmatrix}$$

The projection matrix P is

$$P=A(A^TA)^{-1}A^T=\frac{1}{24}\begin{bmatrix}-2&0\\1&1\\0&2\end{bmatrix}\begin{bmatrix}5&-1\\-1&5\end{bmatrix}\begin{bmatrix}-2&1&0\\0&1&2\end{bmatrix}=\frac{1}{6}\begin{bmatrix}5&-2&1\\-2&2&2\\1&2&5\end{bmatrix}$$

$$\vec{b}_W=P\vec{b}=\frac{1}{6}\begin{bmatrix}5&-1&-2\\-1&5&-2\\-2&-2&2\end{bmatrix}\begin{bmatrix}2\\1\\3\end{bmatrix}=\frac{1}{6}\begin{bmatrix}11\\4\\19\end{bmatrix}$$