姓名: SOLUTION 葉均承 應數一線性代數

學號: _____ **Quiz 8** 考試日期: 2022/05/05

不可使用手機、計算器，禁止作弊!

1. Find the change-of-coordinates matrix from B to B' and from B' to B, indicate which is which, and use it to find the coordinate vector $\vec{v}_{B'}$ with

$$B = (x^2, x, 1), \quad B' = (x^2 - 2x, 2x^2 - 2x + 1, x + 1), \quad \vec{v} = 3x^2 - 5x + 1$$

Answer: $C_{BB'} = \begin{bmatrix} -3 & -2 & 2 \\ 2 & 1 & -1 \\ -2 & -1 & 2 \end{bmatrix}$, $C_{B'B} = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $\vec{v}_B = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$.

Let's rewrite both bases as its coordinate vector relative to B. Therefore, the problem is requesting to change the base from $\tilde{B}$ to $\tilde{B}'$

$$\tilde{B} = ([1,0,0],[0,1,0],[0,0,1]), \ \tilde{B}' = ([1,-2,0],[2,-2,1],[0,1,1]), \ \vec{\tilde{v}} = [3,-5,1]$$

$$\left[M_{\tilde{B}'} \mid M_{\tilde{B}}\right] = \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ -2 & -2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & -2 & 2 \\ 0 & 1 & 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & -2 & -1 & 2 \end{array}\right] = \left[I \mid C_{\tilde{B}\tilde{B}'}\right]$$

Hence

$$C_{\tilde{B}\tilde{B}'} = \begin{bmatrix} -3 & -2 & 2 \\ 2 & 1 & -1 \\ -2 & -1 & 2 \end{bmatrix}, \ C_{\tilde{B}'\tilde{B}} = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\vec{v}_{B'} = \vec{\tilde{v}}_{\tilde{B}'} = C_{\tilde{B}\tilde{B}'}\vec{\tilde{v}}_{\tilde{B}} = \begin{bmatrix} -3 & -2 & 2 \\ 2 & 1 & -1 \\ -2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$