姓名: SOLUTION 葉均承 應數一線性代數

學號: ________________

Quiz 9

考試日期: 2022/05/11

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Find the matrix representations $R_{B,B}$, $R_{B',B'}$ and an invertible C such that $R_{B',B'} = C^{-1}R_{B,B}C$ for the given linear transformation T.
   $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined as reflection of $\mathbb{R}^3$ through the plane $2x + 3y + z = 0$;
   $B = ([2,3,1],[0,1,-3],[1,0,-2])$, $B' = E$.

   $C_{B,B'}=\begin{bmatrix}2&0&1\\3&1&0\\1&-3&-2\end{bmatrix}$, $C_{B',B}=\frac{1}{14}\begin{bmatrix}2&3&1\\-6&5&-3\\10&-6&-2\end{bmatrix}$, $R_{B',B'}=\frac{1}{7}\begin{bmatrix}3&-6&-2\\-6&-2&-3\\-2&-3&6\end{bmatrix}$ and $R_{B,B}=\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}$.

   Is C=$C_{B,B'}$ or $C_{B',B}$? $C_{B'B}$.

Since $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined as reflection of $\mathbb{R}^3$ through the plane $2x + 3y + z = 0$,

T has eigenvectors $\begin{bmatrix}2\\3\\1\end{bmatrix}, \begin{bmatrix}0\\1\\-3\end{bmatrix}, \begin{bmatrix}1\\0\\-2\end{bmatrix}$ corresponding to eigenvalues $-1, 1, 1$. Thus

$$T(\begin{bmatrix}x\\y\\z\end{bmatrix}) = \begin{bmatrix}2&0&1\\3&1&0\\1&-3&-2\end{bmatrix}\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}2&0&1\\3&1&0\\1&-3&-2\end{bmatrix}^{-1}\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{7}\begin{bmatrix}3&-6&-2\\-6&-2&-3\\-2&-3&6\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}$$

We have

$$R_{B',B'} = R'_B = R_E = \frac{1}{7}\begin{bmatrix}3&-6&-2\\-6&-2&-3\\-2&-3&6\end{bmatrix}$$

By $C_{B,B'} = M_{B'}^{-1}M_B = M_E^{-1}M_B = I^{-1}M_B = M_B$,

$$C_{B,B'} = \begin{bmatrix}2&0&1\\3&1&0\\1&-3&-2\end{bmatrix}$$

$$C = C_{B',B} = C_{B,B'}^{-1} = \begin{bmatrix}2&0&1\\3&1&0\\1&-3&-2\end{bmatrix}^{-1} = \frac{1}{14}\begin{bmatrix}2&3&1\\-6&5&-3\\10&-6&-2\end{bmatrix}$$

Since

$$T([2,3,1]) = [-2,-3,-1],\ T([0,1,-3]) = [0,1,-3],\ T([1,0,-2]) = [1,0,-2],$$

$$\because [M_B \mid M_{T(B)}] \sim [I \mid R_B] \quad \therefore \left[\begin{array}{ccc|ccc}2&0&1&-2&0&1\\3&1&0&-3&1&0\\1&-3&-2&-1&-3&-2\end{array}\right] \sim \left[\begin{array}{ccc|ccc}1&0&0&-1&0&0\\0&1&0&0&1&0\\0&0&1&0&0&1\end{array}\right]$$

Thus

$$R_{B,B} = R_B = \begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$