Quiz 11

不可使用手機、計算器，禁止作弊!

1. Let $T : P_2 \to P_3$ be defined by $T(p(x)) = (x-1)p(x+2)$, the ordered basis for P_2 is $B = (x^2, x, 1)$ and the ordered basis for P_3 is $B' = (x^3, x^2, x, 1)$. Fine the standard matrix representation A of T relative to the ordered bases B and B'.

Answer: (a) A $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 1 & 1 \\ -4 & -2 & -1 \end{bmatrix}$

(b) $T(x^2 + 5x - 2) =$ $x^3 + 8x^2 + 3x + 12$

(c) The $ker(T) =$ $\{0\}$

Solution：

$$T(x^2) = (x-1)(x+2)^2 = x^3 + 3x^2 - 4,$$
$$T(x) = (x-1)(x+2) = x^2 + x - 2,$$
$$T(1) = x - 1$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 1 & 1 \\ -4 & -2 & -1 \end{bmatrix}, \ rref(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

By the $rref(A)$, we find the $ker(T)_B = \{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \}$, i.e. $ker(T) = \{0x^2 + 0x + 0 = 0\}$

Let $p(x) = x^2 + 5x - 2$, $p(x)_B = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$.

$$T(p)_{B'} = Ap_B = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 1 & 1 \\ -4 & -2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 3 \\ 12 \end{bmatrix}$$

$T(p) = x^3 + 8x^2 + 3x + 12$

2. Circle each of the following True or False. Please give a counterexample （反例） for the false statement and give an explain （解釋） for the true statement.

(a) **True** False A linear transformation $T : V \to V'$ carries a pair $\vec{v}, \ -\vec{v}$ in V into a pair $\vec{v}', \ -\vec{v}'$ in V'

Solution：

Let $\vec{v}' = T(\vec{v})$

$$T(-\vec{v}) = T((-1)\vec{v}) = (-1)T(\vec{v}) = -T(\vec{v}) = -\vec{v}'$$

(b) **True** False The function $T_{0'} : V \to V'$ defined by $T_{0'}(\vec{v}) = \vec{0}'$, the zero vector of V', for all $\vec{v}$ in V is a linear transformation.

Solution：

For any $\vec{v}, \ \vec{u} \in V$,

$$T_{0'}(\vec{v}) + T_{0'}(\vec{u}) = \vec{0}' + \vec{0}' = \vec{0}' = T_{0'}(\vec{v} + \vec{u})$$

For any $\vec{v} \in V$, $r \in \mathbb{R}$

$$rT_{0'}(\vec{v}) = r\vec{0}' = \vec{0}' = T_{0'}(r\vec{v})$$