不可使用手機、計算器，禁止作弊!

1. Determinant whether the given 4 points lie in a plane in $\mathbb{R}^3$. If so, find its area. If not, find its volume.

$$A(1,1,1), B(1,4,3), C(2,5,8), D(-1,2,4)$$

Answer: ✗ $ABCD$ are coplanar(共平面), and the area is ___**N/A**___ .

✓ $ABCD$ are NOT coplanar(共平面), and the volume is ___**16.5**___ .

Solution：
$\overrightarrow{AB} = [0,3,2], \overrightarrow{AC} = [1,4,7], \overrightarrow{AD} = [-2,1,3]$

$$\begin{vmatrix} 0 & 3 & 2 \\ 1 & 4 & 7 \\ -2 & 1 & 3 \end{vmatrix} = -33 \neq 0$$

So the points are not coplanar and the volume of the Parallelepiped (平行六面體) formed by coterminous (相鄰邊) edges $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ is 33.

The volume of a tetrahedron (四面體) $ABCD$ formed by coterminous (相鄰邊) edges $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ is

$$\frac{\text{volume of the Parallelepiped}}{6} = \frac{33}{6} = 16.5$$

2. Circle each of the following True or False. Please give a counterexample （反例）for the false statement and give an explain （解釋）for the true statement.

True   **False**   The determinant of 3×3 matrix is zero if the points in $\mathbb{R}^3$ given by the rows of the matrix lie in a plane.

Solution：

$A:(1,0,0),\ B=(0,1,0),\ C=(0,0,1)$ are lie in the plane $x+y+z=1$.

However,

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$