

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Find all possible scalar c such that the vector $-3\vec{i} + 2\vec{j} + c\vec{k}$ is in the span of $2\vec{i} - \vec{j} - \vec{k}$ and $\vec{i} + 3\vec{k}$.

Answer: $c = 5$.

We can rewrite the question as the following:

Find all possible scalar c such that the vector $\vec{v} = [-3, 2, c]$ is in the span of $\vec{a} = [2, -1, -1]$ and $\vec{b} = [1, 0, 3]$.

Since $\vec{b} = [1, 0, 3]$ does not include $\vec{j} (= [0, 1, 0])$, the $\vec{a} = [2, -1, -1]$ has to respond for the $[0, 1, 0]$ part of $\vec{v} = [-3, 2, c]$. Therefore, the scalar coefficient of $\vec{a} = [2, -1, -1]$ is -2 . The vector $\vec{b} = [1, 0, 3]$ is parallel to $\vec{v} - (-2)\vec{a} = [-3, 2, c] - (-2)[2, -1, -1] = [1, 0, c - 2]$. Hence, we have $3 \times 1 = c - 2$, and then $c = 5$.

2. Let \vec{u} be any vector in \mathbb{R}^n , and let r and s be any scalars in \mathbb{R} . Please prove the following property.

$$(r + s)\vec{v} = r\vec{v} + s\vec{v}.$$

Check example 4 from 1-1.