姓名: SOLUTION 葉均承 應數一線性代數

學號: ______________ **Quiz 2** 考試日期: 2022/09/20

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Compute the indicated quantity (量), if it is defined.

$$A = \begin{bmatrix} 4 & 1 & -2 \\ 1 & -1 & 3 \end{bmatrix},\ B = \begin{bmatrix} 2 & 0 & 3 \\ -1 & 4 & 2 \end{bmatrix},\ C = \begin{bmatrix} 2 & -1 \\ 0 & 3 \\ -1 & 2 \end{bmatrix}$$

(a) $(2A)(-B) =$ undefined. (b) $(2A)(-C) = \begin{bmatrix} -20 & 10 \\ 2 & -4 \end{bmatrix}$.

```
octave:1> A=[4 1 -2; 1 -1 3]
A =

   4   1  -2
   1  -1   3

octave:2> B=[2 0 3;-1 4 2]
B =

   2   0   3
  -1   4   2

octave:3> C=[2 -1; 0 3; -1 2]
C =

   2  -1
   0   3
  -1   2
```

```
octave:4> (2*A)*(-B)
error: operator *: nonconformant arguments (op1 is 2x3, op2 is 2x3)
octave:5> (2*A)*(-C)
ans =

  -20   10
    2   -4
```

2. Write down the expression for the (i,j) element of AB in terms of only the elements of matrices A and B.

Given A is a $m \times n$ matrix, written as $[a_{ij}]_{m\times n}$, and B is a $n \times s$ matrix, written as $[b_{ij}]_{n\times s}$. Then $C = AB$ is a $m \times s$ matrix, written as $[c_{ij}]_{m\times s}$.

The (i,j) element of AB is c_{ij}, where

$$c_{ij} = \begin{bmatrix} a_{i1} & a_{i2} & \dots & a_{in} \end{bmatrix} \begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{nj} \end{bmatrix} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

Circle each of the following True or False and then give a counterexample（反例）for the false statement.

3. True **False** The magnitude of $\vec{v} + \vec{w}$ must be at least as large as the magnitude of either $\vec{v}$ or $\vec{w}$ in $\mathbb{R}^n$.

   Let $\vec{v} = [1, 0]$, $\vec{w} = [-1, 0]$, then $\|\vec{v} + \vec{w}\| = 0 < 1 = \|\vec{v}\| = \|\vec{w}\|$

4. True **False** There are exactly two unit vectors perpendicular (垂直) to any given nonzero vectors in $\mathbb{R}^n$.

   For $n = 3$, $\vec{e}_1 = [1, 0, 0]$ is perpendicular to every vector in $S = sp(\vec{e}_2, \vec{e}_3) = sp([0, 1, 0], [0, 0, 1])$.

5. True **False** The dot product od a vector with itself yields the magnitude of the vector.

   Should be the "SQUARE" of the magnitude of the vector.