姓名: SOLUTION　　　　　　　　　　　　　　　　葉均承　　應數一線性代數

學號: ____________　　　　**Quiz 3**　　　　考試日期: 2022/09/27

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Prove that the given relation holds for all matrices A and scalars $r, s \in \mathbb{R}$.

$$(r+s)A = rA + sA$$

1-3 #27 (課本有解答)

2. Determine whether the vector $\vec{b}$ is in the span of the vectors $\vec{v_i}$. If so, write $\vec{b}$ into the linear combination form.

p.s. Please solve the problem with the corresponding augmented matrix. Also mark the row-echlon form and reduced row-echlon form of the augmented matrix.

$$\vec{b} = \begin{bmatrix} -3 \\ -1 \\ 5 \end{bmatrix}, \vec{v_1} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix}$$

Solution:

augmented matrix: $\begin{bmatrix} 0 & 1 & 3 & -3 \\ 2 & 4 & 5 & -1 \\ 4 & -2 & 3 & 5 \end{bmatrix}$, reduced row-echlon form: $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

Yes! the vector $\vec{b}$ is in the span of the vectors $\vec{v_i}$.

$$\vec{b} = 2 \cdot \vec{v_1} + 0 \cdot \vec{v_2} - \vec{v_3}$$

$$\begin{bmatrix} -3 \\ -1 \\ 5 \end{bmatrix} = 2 \cdot \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} + 0 \cdot \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} - \begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix}$$

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octave:1> A=[0 1 3 -3; 2 4 5 -1;4 -2 3 5]
A =

   0   1   3  -3
   2   4   5  -1
   4  -2   3   5

octave:2> rref(A)
ans =

   1   0   0   2
   0   1   0   0
   0   0   1  -1
```