姓名: SOLUTION　　葉均承　　應數一線性代數

學號: _____　　**Quiz 4**　　考試日期: 2022/10/05

不可使用手機、計算器，禁止作弊!

1. Determine all values of the b_i that make the linear system $\begin{cases} 6x_1 + 9x_2 = b_1 \\ 8x_1 + 13x_2 = b_2 \end{cases}$ consistent.

   Answer: ☐ the linear system is always inconsistent.

   ☑ the linear system is consistent when the b_i satisfy for all kind of $b_i \in \mathbb{R}$.

$$\left[\begin{array}{cc|c} 6 & 9 & b_1 \\ 8 & 13 & b_2 \end{array}\right] \sim \left[\begin{array}{cc|c} 6 & 9 & b_1 \\ 0 & 1 & b_2 - \frac{4}{3}b_1 \end{array}\right]$$

   Therefore, we know the coefficient matrix can be row-reduced to the identity matrix. Hence, the linear system

2. (a) Find the inverse of the matrix A, if it exists, and (b) express the inverse matrix as a product of elementary matrices. $A = \begin{bmatrix} 6 & 9 \\ 8 & 13 \end{bmatrix}$

   Answer: (a) $A^{-1} = \begin{bmatrix} \frac{13}{6} & \frac{-3}{2} \\ \frac{-4}{3} & 1 \end{bmatrix}$, (b) $A^{-1} = \begin{bmatrix} 1/6 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -9 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -4/3 & 1 \end{bmatrix}$