姓名: SOLUTION 葉均承 應數一線性代數

學號: ________________ **Quiz 5** 考試日期: 2022/10/12

不可使用手機、計算器，禁止作弊!

1. Determine whether the given subset is a subspace of $\mathbb{R}^3$. Please give reasons to support your answer.

$$\{[2x, x+2y, y+1] | x, y \in \mathbb{R}\}$$

Circle the answer: (Yes / No), and write your reason below.

Answer: There's many ways to prove that W is a subspace of $\mathbb{R}^3$. Just lists 4 of them in here.

solution 1:
Assume W is a subspace of $\mathbb{R}^3$. Pick $x = 0, y = 3$ and we know $\vec{v} = [0, 6, 4] \in W$. Pick $r = 2 \in R$.
There must exists $a, b \in \mathbb{R}$ such that $r\vec{v} = 2[0, 6, 4] = [0, 12, 8] = [2a, a + 2b, b + 1] \in W$
It is easy to know that $a = 0$ and we can't find a $b \in \mathbb{R}$ so that $[0, 2b, b + 1] = [0, 12, 8]$. Therefore, W is NOT a subspace of $\mathbb{R}^3$.

solution 2:
For any $\vec{v}, \vec{u} \in W$. Let $\vec{v} = [2x, x + 2y, y + 1], \vec{u} = [2p, p + 2q, q + 1]$, $x, y, p, q \in \mathbb{R}$.
Since $\vec{v}+\vec{u} = [2x, x+2y, y+1]+[2p, p+2q, q+1] = [2(x+p), (x+p)+2(y+q), (y+q)+2]$, we have $(\vec{v} + \vec{u}) \notin W$.
Hence W is NOT a subspace of $\mathbb{R}^n$

solution 3:
For any $\vec{v} \in W$, any $r \in R$. Let $\vec{v} = [2x, x + 2y, y + 1]$, $x, y \in \mathbb{R}$.
Since $r\vec{v} = r[2x, x + 2y, y + 1] = [2(rx), (rx) + 2(ry), (ry) + r] \notin W$.
Hence W is NOT a subspace of $\mathbb{R}^n$

solution 4:
For any $\vec{v}, \vec{u} \in W$, any $r, s \in R$. Let $\vec{v} = [2x, x+2y, y+1], \vec{u} = [2p, p+2q, q+1]$, $x, y, p, q \in \mathbb{R}$.
Since $r\vec{v} + s\vec{u} = r[2x, x + 2y, y + 1] + s[2p, p + 2q, q + 1] = [2(rx + sp), (rx + sp) + 2(ry + sq), (ry + sq) + (r + s)]$, we have $(r\vec{v} + s\vec{u}) \notin W$.
Hence W is NOT a subspace of $\mathbb{R}^n$

2. Find a basis for the solution set of the given homogeneous linear system.

$$\begin{cases} 3x_1 + \ \ x_2 + \ \ x_3 = 0 \\ 6x_1 + 2x_2 + 2x_3 = 0 \\ -9x_1 - 3x_2 - 3x_3 = 0 \end{cases}$$

Answer: the basis for the solution set is $\left\{ \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix} \right\}$

Let $A = \begin{bmatrix} 3 & 1 & 1 \\ 7 & 2 & 2 \\ -9 & -3 & -3 \end{bmatrix}$, and $H = rref(A) = \begin{bmatrix} 1 & 1/3 & 1/3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Assume $x_2 = r, x_3 = s$, plug into $[H|0]$. We have

$$x_1 + \frac{r}{3} + \frac{s}{3} = 0,\ 0 = 0,\ 0 = 0.$$

Hence, $x_1 = -\frac{r}{3} - \frac{s}{3}, x_2 = r, x_3 = s$. We have solution set

$$\left\{ \frac{r}{3} \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} + \frac{s}{3} \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix} \middle| r, s \in \mathbb{R} \right\} = sp\left(\begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix} \right)$$