Quiz 6

不可使用手機、計算器，禁止作弊!

1. Let the subspace $W = sp([1,-3,2],[2,1,3],[4,9,5])$ in $\mathbb{R}^3$.

   (a) Find $dim(W)$.

   (b) Find a basis for W.

   (c) Is $dim(W) = 3$? If not, enlarge the basis you get in (b) to be a basis for $\mathbb{R}^3$.

Answer:

Let $A = \begin{bmatrix} 1 & 2 & 4 & 1 & 0 & 0 \\ -3 & 1 & 9 & 0 & 1 & 0 \\ 2 & 3 & 5 & 0 & 0 & 1 \end{bmatrix}$, and $H = rref(A) = \begin{bmatrix} 1 & 0 & -2 & 0 & -3/11 & 1/11 \\ 0 & 1 & 3 & 0 & 2/11 & 3/11 \\ 0 & 0 & 0 & 1 & -1/11 & -7/11 \end{bmatrix}$

Since the pivots are in the 1^{st}, 2^{nd} and 4^{th} column of H, we have:

1. The $dim(W) = 2$.
2. A basis for W is $\{[1,-3,2],[2,1,3]\}$.
3. A requested basis for $\mathbb{R}^3$ is $\{[1,-3,2],[2,1,3],[1,0,0]\}$.

Circle each of the following True or False and then give a counterexample（反例）for the false statement.

2. **True** False If a subset of two vectors in $\mathbb{R}^2$ spans $\mathbb{R}^2$, then the subset is linearly independent.

3. True **False** Every independent subsets of $\mathbb{R}^n$ is a subset of every basis for $\mathbb{R}^n$.

For $n = 3$, $\{[1,0,0],[0,2,0]\}$ is a linearly independent subset of $\mathbb{R}^3$. $\{[1,0,0],[0,1,0],[0,0,1]\}$ is a basis for $\mathbb{R}^3$. However, $\{[1,0,0],[0,2,0]\}$ is not a subset of $\{[1,0,0],[0,1,0],[0,0,1]\}$.

4. True **False** If a linear system $A\vec{x} = \vec{0}$ has only the trivial solution, then $A\vec{x} = \vec{b}$ has a unique solution for every column vector $\vec{b}$ with the appropriate number of components.

Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 2 \end{bmatrix}, \; rref(A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Hence $A\vec{x} = \vec{0}$ has only the trivial solution, but $A\vec{x} = \vec{b}$ has NO solution when

$$\vec{b} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}$$