不可使用手機、計算器，禁止作弊!

1. Let F bet he set of all real-valued functions on a (nonempty) set S; that is , let F be the set of all functions mapping S into $\mathbb{R}$. For $f, g \in F$, let the sum $f \oplus g$ of two functions f and g in F, and for any scalar r, let scalar multiplication be defined below. Is this set a vector space?

$$(f \oplus g)(x) = 2f(x) + 2g(x) \text{ for all } x \in S$$
$$(r \otimes f)(x) = rf(x) - r \text{ for all } x \in S$$

   a. Is this set a vector space? _____
   Hint: Show by verifying the closed under two operations, A1-A4 and S1-S4.

   b. What is the zero vector in this vector space?
   Answer: the zero vector is _____, for any functions f, the $-f$ is _____

2. Consider the set $\mathbb{R}^2$, with the addition defined by $[x, y] \oplus [a, b] = [x + a + 1, y + b]$, and with scalar multiplication defined by $r \otimes [x, y] = [rx + r - 1, ry]$.

   a. Is this set a vector space? _____
   Hint: Show by verifying the closed under two operations, A1-A4 and S1-S4.

   b. What is the zero vector in this vector space? *Hint:* The zero vector will NOT be the vector $[0, 0]$.
   Answer: the zero vector is _____, for any vectors [x,y], the -[x,y] is _____