姓名: SOLUTION 葉均承 應數一線性代數

學號: ________________ **Quiz 8** 考試日期: 2022/11/01

不可使用手機、計算器，禁止作弊!

1. Let $T([x, y, z]) = [y - z, 2x + z, -x + 2y + z]$ an invertible linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$. Find $T^{-1}([5, -3, 2])$.

   Answer: $T^{-1}([5, -3, 2]) =$ $\frac{-1}{7}[1, 16, 19]$

   Solution：

   Let A is the standard matrix representation of T.

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix},\ T([x, y, z]) = \left(\begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right)^T$$

   The standard matrix representation of T^{-1} is A^{-1}

$$A^{-1} = \frac{-1}{7} \begin{bmatrix} -2 & -3 & 1 \\ -3 & -1 & -2 \\ 4 & -1 & -2 \end{bmatrix}$$

$$T^{-1}([5, -3, 2]) = (A^{-1} \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix})^T = \frac{-1}{7}[1, 16, 19]$$

2. If $B = (\vec{b}_1, \vec{b}_2, ..., \vec{b}_n)$ is a basis for $\mathbb{R}^n$ and T and T' are linear transformations mapping $\mathbb{R}^n$ into $\mathbb{R}^m$. Prove that $T(\vec{x}) = T'(\vec{x})$ for all $\vec{x} \in \mathbb{R}^n$ if $T(\vec{b}_i) = T'(\vec{b}_i)$ for $i = 1, 2, ..., n$.

Solution：

Since $B = (\vec{b}_1, \vec{b}_2, ..., \vec{b}_n)$ is a basis for $\mathbb{R}^n$, for all $\vec{x} \in \mathbb{R}^n$, there exist $r_1, r_2, ..., r_n \in \mathbb{R}$ such that

$$\vec{x} = r_1\vec{b}_1 + r_2\vec{b}_2 + ... + r_n\vec{b}_n$$

Because T and T' are linear transformations, we have

$$\begin{aligned} T(\vec{x}) &= T(r_1\vec{b}_1 + r_2\vec{b}_2 + ... + r_n\vec{b}_n) \\ &= T(r_1\vec{b}_1) + T(r_2\vec{b}_2) + ... + T(r_n\vec{b}_n) \\ &= r_1T(\vec{b}_1) + r_2T(\vec{b}_2) + ... + r_nT(\vec{b}_n) \\ &= r_1T'(\vec{b}_1) + r_2T'(\vec{b}_2) + ... + r_nT'(\vec{b}_n) \\ &= T'(r_1\vec{b}_1) + T'(r_2\vec{b}_2) + ... + T'(r_n\vec{b}_n) \\ &= T'(r_1\vec{b}_1 + r_2\vec{b}_2 + ... + r_n\vec{b}_n) \\ &= T'(\vec{x}) \end{aligned}$$